

Gravitational Waves from Q -ball Formation

Takeshi Chiba

*Department of Physics, College of Humanities and Sciences,
Nihon University, Tokyo 156-8550, Japan*

Kohei Kamada

*Department of Physics, Graduate School of Science,
The University of Tokyo, Tokyo 113-0033, Japan and*

*Research Center for the Early Universe (RESCEU),
Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan*

Masahide Yamaguchi

*Department of Physics and Mathematics,
Aoyama Gakuin University, Sagamihara 229-8558, Japan*

(Dated: December 1, 2010)

Abstract

We study the detectability of the gravitational waves (GWs) from the Q -ball formation associated with the Affleck-Dine (AD) mechanism, taking into account both the dilution effects due to Q -ball domination and to finite temperature. The AD mechanism predicts the formation of nontopological solitons, Q -balls, from which GWs are generated. Q -balls with large conserved charge Q can produce a large amount of GWs. On the other hand, the decay rate of such Q -balls is so small that they may dominate the energy density of the Universe, which implies that GWs are significantly diluted and that their frequencies are redshifted during the Q -ball dominated era. Thus, the detectability of the GWs associated with the formation of Q -balls is determined by these two competing effects. We find that there is a finite but small parameter region where such GWs may be detected by future detectors such as DECIGO or BBO, only in the case when the thermal logarithmic potential dominates the potential of the AD field. Otherwise GWs from Q -balls would not be detectable even by these futuristic detectors: $\Omega_{\text{GW}}^0 < 10^{-21}$. Unfortunately, for such parameter region the present baryon asymmetry of the Universe can hardly be explained unless one fine-tunes A -terms in the potential. However the detection of such a GW background may give us an information about the early Universe, for example, it may suggest that the flat directions with $B - L = 0$ are favored.

PACS numbers: 98.80.Cq ; 04.30.Db

I. INTRODUCTION

Primordial gravitational waves (GWs) provide us with a lot of important information about the early Universe because the interaction of GWs with matter is very weak and they carry the memory of cosmic history during and after inflation [1]. One of the main sources of such primordial GWs is cosmic inflation [2], during which stochastic GWs are generated with a nearly scale-invariant spectrum with frequencies ranging from 10^{-15} Hz to 10^5 Hz, depending on inflation models. Though the amplitude is typically very small, next-generation gravitational detectors like DECIGO [3] and BBO [4] as well as indirect observations through the B -modes in the cosmic microwave background (CMB) anisotropy [5] might be able to detect such GWs. The detection of such stochastic GWs can reveal the history of the early Universe such as the changes of the number of the massless degree of freedom [6], the reheating temperature [7], the neutrino decoupling [8], the lepton asymmetry [9], and so on. Additional GWs can also be generated from cosmological mechanisms like (local/global) phase transitions [10] and preheating after inflation [11, 12] or astrophysical origins like the collapse of Population III stars [13].

Recently, yet another interesting mechanism to produce GWs was proposed in Ref. [14], in which it is shown that significant GWs are emitted during the Q -ball formation associated with the Affleck-Dine (AD) mechanism of baryogenesis. In this scenario, the angular momentum, the rotation around the origin of scalar fields that carry the baryon or lepton number, is dynamically generated, which implies that the baryon or lepton asymmetry is produced. When initially almost homogeneous scalar fields start to rotate, they suffer from spatial instabilities, and their fluctuations begin to grow if the scalar field potential driving the field rotation is flatter than the quadratic potential. Such fluctuations finally settle down into nontopological solitons called Q -balls, whose existence and stability are guaranteed by a conserved charge, Q , associated with a global symmetry [15]. Since the formation process of such Q -balls is inhomogeneous and not spherical, GWs can be generated during the formation.

In order to calculate the present amplitude of such GWs, one has to estimate not only the amount of GWs at the formation of Q -balls but also the dilution factor during the cosmic history after the production of the GWs. As shown in Ref. [14], the energy density of the GWs at the Q -ball formation is proportional to some powers of the field value of the Affleck-Dine condensate, which implies that the initial energy density of the GWs becomes large if the typical charge Q of Q -balls is large. On the other hand, the lifetime of Q -balls becomes longer for larger Q because the temperature at the decay of Q -balls is typically proportional to the inverse square root of the charge Q [16]. Therefore, Q -balls with large Q can quickly dominate the energy density of the Universe and hence dilute the GWs significantly. Thus, the maximum value of the present amplitude of such GWs is determined by the balance of the above two competing effects. In Refs. [14], in order to avoid the large dilution by the Q -ball dominance, Q -balls are assumed to decay quickly by some artificial effects, which are unnatural in the context of the minimal supersymmetric standard model (MSSM). Then, in this paper, we reconsider the decay of Q -balls without such effects and calculate the amplitude of GWs from Q -balls taking into account of the dilution factor correctly, which results in dramatic changes in the present amplitude of the GWs from the Q -ball formation.

The properties of the Affleck-Dine mechanism [17] and the subsequent formation of Q -balls depend on the supersymmetry (SUSY) breaking mechanism [18, 19]. This is mainly because the effective potential of a flat direction and the gravitino mass are quite different for

different mediation mechanisms. There are many types of Q -balls such as gauge-mediation type [18, 20], gravity-mediation type [19, 21], new type [22], delayed type [23], and so on.

Moreover, the effective potential of a flat direction consists of thermal terms as well as the zero-temperature terms. The properties of Q -balls are quite different depending on which term dominates the potential energy. When thermal (logarithmic) effects [24] dominate the effective potential, the formed Q -balls, called thermal log type [23], have an interesting property. Namely, while the energy density of other types of Q -balls decreases like matter, that of the thermal log type Q -ball decreases at least as rapid as radiation. This is because the thermal logarithmic potential itself also decreases with the cosmic expansion while the number of Q -balls in a comoving volume does not change. Hence, this type of Q -balls cannot dominate the energy density of the Universe and do not dilute GWs. This is favorable for the detection of the GWs from the Q -ball formation because the dilution during the Q -ball dominated era is the main obstacle for the detection. In fact, as is explicitly shown in Appendix B, GWs from the Q -ball formation in the zero-temperature potential may not be detectable even by the next-generation detectors. Therefore, in this paper, we concentrate on the thermal log type Q -balls and estimate the present amplitudes and frequencies of the GWs at the formation of such Q -balls. We show that such GWs may be detected by the next-generation gravitational detectors like DECIGO and BBO if particular conditions of reheating temperature, the initial field value of the AD field, gravitino mass and messenger mass are realized in the gauge-mediated SUSY-breaking model. However, we also find that such a condition spans a very small region in the parameter space. Moreover, we also find that it is difficult to explain the present baryon asymmetry for such a parameter region, unless one fine-tunes the CP -violating A -terms in the potential.

The paper is organized as follows. In Sec. II, we give a brief review of the dynamics of the Affleck-Dine baryo/leptogenesis and the properties of the subsequently produced Q -balls, particularly paying attention to the decay of Q -balls. In Sec. III, we evaluate how many GWs are produced at the formation of Q -balls and are diluted during the cosmic history, which yield the present amplitude of the GWs from the Q -ball formation. Section IV is devoted to discussion and conclusions. We concentrate on thermal log type Q -balls in the main body of the paper. The cases with other types of Q -balls, in which the zero-temperature effect (and the negative thermal log effect) dominate the effective potential are discussed in Appendix B.

II. AFFLECK-DINE MECHANISM AND Q -BALLS

In this section, we briefly review the AD mechanism, and the formation and the fate of the nontopological solitons, Q -balls. In supersymmetric theories, there are many flat directions along which the scalar potentials become flat in the global SUSY limit, and hence such scalar fields can easily acquire large field values. The flat directions in the MSSM consist of squarks, sleptons, and Higgs fields, and are parameterized by composite gauge-invariant monomial operators such as $\bar{u}d\bar{d}$ and $LL\bar{e}$ [25]. Thus, flat directions can carry baryon (B) or lepton (L) charges in general. This is the reason why the AD mechanism can be one of the powerful models of baryogenesis. We will see its details in the following subsection.

The dynamics of a flat direction can be expressed in terms of a scalar field Φ (the AD field). We consider the dynamics of a flat direction Φ , taking into account the finite temperature effects since these effects play important roles in the formation and the evolution of Q -balls.

A. Affleck-Dine mechanism

The scalar potential vanishes along flat directions in the global SUSY limit. However, it is lifted by the SUSY-breaking effects, which depend on the SUSY-breaking mechanism. Moreover it is also lifted by a nonrenormalizable operator in the superpotential,

$$W = \frac{\Phi^n}{nM^{n-3}}, \quad (1)$$

where M is a cutoff scale for an interaction and n is an integer, which depends on a flat direction Φ . For example, $n = 6$ if Φ parameterizes the $\bar{u}d\bar{d}$ flat direction since $(\bar{u}d\bar{d})^2$ is the lowest order nonrenormalizable operator.

In the gravity or anomaly mediated SUSY-breaking scenario, the flat directions acquire their masses [19],

$$V_{\text{grav}} \simeq m_\phi^2 \left[1 + K \log \left(\frac{|\Phi|^2}{M_G^2} \right) \right] |\Phi|^2, \quad (2)$$

where $m_\phi \sim \text{TeV}$ is the soft SUSY-breaking mass, $M_G = 1/\sqrt{8\pi G}$ is the reduced Planck mass, and K is a numerical coefficient coming from one-loop corrections. The sign of K depends on the details of the loop effects: K can be positive if the top quark loop effects are the dominant contribution, which is realized when the top Yukawa coupling is order unity. On the other hand, K is negative with $K = (-0.01 \sim -0.1)$ when the gaugino loop effects are dominant [19]. As shown later, Q -balls are formed when K is negative.

In the gauge-mediated SUSY-breaking model, on the other hand, the potential along the flat direction is lifted as [26],

$$V_{\text{gauge}} \simeq \begin{cases} m_\phi^2 |\Phi|^2 & (|\Phi| \ll M_S), \\ M_F^4 \left(\log \frac{|\Phi|^2}{M_S^2} \right)^2 & (|\Phi| \gg M_S), \end{cases} \quad (3)$$

where M_F is the SUSY-breaking scale and $M_S = M_F^2/m_\phi$ is the messenger mass. The allowed parameter range of M_F is $10^4 \text{GeV} \lesssim M_F \lesssim 10^{10} \text{GeV}$ [26]. The upper bound comes from the condition that the gravity effects should not be so strong and the lower bound comes from the condition that the SUSY breaking scale should be larger than the electroweak scale. The shape of the potential can be understood by noting that for large $|\Phi| > M_S$, the supersymmetry-breaking mass terms are suppressed by a factor of $M_S^2/|\Phi|^2$ and hence the scalar potential becomes constant at $|\Phi| > M_S$. In addition to V_{gauge} , there also exists the gravity effects,

$$V_{\text{grav2}} \simeq m_{3/2}^2 \left[1 + K \log \left(\frac{|\Phi|^2}{M_G^2} \right) \right] |\Phi|^2, \quad (4)$$

where $m_{3/2}$ is the gravitino mass ranging from 1 eV to 10 GeV. This term typically dominates over V_{gauge} at large field values.

The scalar potential also contains the contribution from nonrenormalizable operators, called A -term V_A and F -term V_F :

$$V_A = \frac{a_m m_{3/2}}{M^{n-3}} \Phi^n + \text{H.c.}, \quad (5)$$

$$V_F = \frac{|\Phi|^{2n-2}}{M^{2n-6}}, \quad (6)$$

where a_m is a complex parameter and its absolute magnitude is less than order unity. A -term arises from the gravitational interaction between the nonrenormalizable operator and the SUSY-breaking sector.

During both inflation and the inflaton oscillation dominated era, the AD field receives the Hubble induced mass term coming from the gravitational interaction between the AD field and the inflaton :

$$V_{\text{HM}} = -c_H H^2 |\Phi|^2, \quad (7)$$

where c_H is a positive constant of order unity and H is the Hubble parameter. The balance between F -term and the (negative) Hubble induced mass term determines the initial value of the AD field, but these terms are irrelevant for the subsequent dynamics.

In addition to the SUSY-breaking effects and the nonrenormalizable operators, there are other contributions to the scalar potential. When the thermal plasma exists, the AD field receives the finite temperature effects given by [24],

$$V_{\text{thermal}} \sim \begin{cases} h^2 T^2 |\Phi|^2 & (h|\Phi| < T), \\ c\alpha_g^2 T^4 \log\left(\frac{|\Phi|^2}{T^2}\right) & (h|\Phi| > T), \end{cases} \quad (8)$$

where h is the Yukawa or the gauge coupling constant for the corresponding AD field, T is the temperature of the thermal plasma, c is a numerical constant of order unity, and $\alpha_g \equiv g^2/4\pi$ represents the gauge coupling constant. The sign of c depends on the AD field and we assume it to be positive henceforth. The upper term in the right-hand side of Eq. (8) represents the thermal mass from the thermal plasma and the lower one represents the two-loop finite temperature effects coming from the running of the gauge coupling $g(T)$ with the nonzero field value of the AD field. Note that the thermal plasma exists even before the reheating from the inflaton decay. This is because the partial decay of the inflaton before its complete decay generates the thermal plasma as a subdominant component of the Universe. During the inflaton oscillation dominated era, the temperature of the Universe can be expressed as [27]

$$T \simeq A_T^{1/8} \left(\frac{H M_G T_R^2}{A} \right)^{1/4} (\propto a^{-3/8}), \quad (9)$$

where $A \equiv \pi^2 g_*/90$ with g_* being the effective relativistic degrees of freedom. The subscript “ T ” indicates that the parameter is evaluated at the scale higher than 1 TeV. Here we have assumed that reheating from inflaton decay takes place at the temperature higher than 1 TeV. Note that, in the context of MSSM, at the energy scale above ~ 1 TeV, $g_* \simeq 220$, at the energy scale $100 \text{ MeV} \sim 1 \text{ TeV}$, $g_* \simeq 100$, at the energy scale $0.1 \text{ MeV} \sim 100 \text{ MeV}$, $g_* \simeq 10$ and at the energy scale below $\sim 0.1 \text{ MeV}$, $g_* \simeq 4$. The temperatures at the onset of the AD-field oscillation, Q -ball formation and domination, and reheating, which are given in Sec. II C, are higher than the electroweak scale when the amplitude of the GWs is large enough. Thus, we neglect the time variation of the relativistic degrees of freedom before Q -ball formation and reheating and we express A at those epochs as A_T henceforth. On the other hand, the temperatures at Q -ball domination and Q -ball decay can be less than 1 TeV and hence we express A at that time as A_{dom} and A_{dec} , respectively.

Now we consider the dynamics of the AD field. The total effective potential of the AD field V is then given by $V = V_{\text{grav/gauge}} + (V_{\text{grav2}}) + V_A + V_F + V_{\text{thermal}} + V_{\text{HM}}$, and the AD

field obeys the equation of motion,

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{\partial V}{\partial \Phi^*} = 0, \quad (10)$$

where the dot denotes the derivative with respect to the cosmic time, t . The AD field quickly settles down to the potential minimum $|\Phi| \simeq (HM^{n-3})^{1/(n-2)}$, which is determined by the balance between V_{HM} and V_F .

V_{HM} decreases after inflation in response to the decrease of the Hubble parameter and disappears after the reheating of the Universe. Then, the AD field (more precisely its radial component) begins to oscillate around the origin when

$$H_{\text{osc}}^2 = V''(\Phi), \quad (11)$$

where the dash denotes the derivative with respect to $\phi \equiv \sqrt{2}|\Phi|$. Hereafter the subscript “osc” indicates that the parameter or the variable is evaluated at the beginning of the oscillation of the AD field. The field value at which the AD field begins to oscillate is given by $\phi_{\text{osc}} \simeq (H_{\text{osc}} M^{n-3})^{1/(n-2)}$.

The potential of the AD field also contains a phase dependent term, that is, A -term, which rotates the AD field unless the phase component of the AD field, $\theta \equiv \arg[\Phi]$, accidentally sits on the potential valley of the A -term Eq. (5). As a consequence, the orbit of the AD field in the phase space becomes elliptical and its ellipticity is estimated to be $\epsilon \simeq a_m m_{3/2} / V''(\phi_{\text{osc}})$.

If the AD field carries baryon or lepton charge β_c , the angular momentum of the motion in the complex plane of the AD field represents the baryon or lepton number density given by

$$n_B(t_{\text{osc}}) = i\beta_c(\dot{\Phi}^*\Phi - \Phi^*\dot{\Phi}) \simeq \beta_c a_m m_{3/2} \phi_{\text{osc}}^2, \quad (12)$$

which implies that baryon or lepton asymmetry is generated in the Universe.

B. Q -ball formation

Next we consider the Q -ball formation associated with the AD mechanism. Fluctuations around the homogeneous mode feel spatial instabilities and grow nonlinearly during the oscillation of the AD field [18] and eventually form clumpy objects, Q -balls, if $V(\phi)/\phi^2$ has a global minimum at $\phi = \phi_{\text{min}} \neq 0$ [15]. This is because the pressure of the AD field is negative for such a potential [19]. From Eqs. (2), (3), and (8), the condition can be realized when $V_{\text{grav}(2)}$ with negative K , $V_{\text{gauge}}(\phi > M_S)$, or $V_{\text{thermal}}(\phi > hT)$ dominates the potential energy. In fact, it is confirmed numerically that fluctuations develop and go nonlinear to form Q -balls [20, 21, 23]. Their stabilities are guaranteed by global $U(1)$ charge, that is, baryon or lepton charge in our case. In this subsection we briefly review the amplification of the fluctuations of the AD field and the properties of the subsequently produced Q -balls. Here we concentrate on the case when the dynamics of the AD field is driven by the thermal logarithmic potential Eq. (8). In Appendix B, we will comment on the cases when the effective potential is dominated by zero-temperature terms (and a negative thermal log term).

First, let us examine the growing of the fluctuations of the AD field using the linear perturbation analysis. We write the AD field Φ as

$$\Phi = \frac{\phi}{\sqrt{2}} e^{i\theta}, \quad (13)$$

and decompose the radial (ϕ) and the phase (θ) components into their homogeneous parts and perturbations,

$$\phi(\mathbf{x}, t) = \phi(t) + \delta\phi(\mathbf{x}, t), \quad (14)$$

$$\theta(\mathbf{x}, t) = \theta(t) + \delta\theta(\mathbf{x}, t). \quad (15)$$

Once the baryon or the lepton number is fixed, the phase dependent term in the potential is irrelevant. Then, neglecting such terms in the potential, the equations of motion in the flat FRW universe read [18]

$$\ddot{\phi} + 3H\dot{\phi} - \phi\dot{\theta}^2 + V' = 0, \quad (16)$$

$$\ddot{\theta} + 3H\dot{\theta} + 2\frac{\dot{\phi}}{\phi}\dot{\theta} = 0, \quad (17)$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{1}{a^2}\Delta\delta\phi - (2\dot{\theta}\phi\delta\dot{\theta} + \dot{\theta}^2\delta\phi) + V''\delta\phi = 0, \quad (18)$$

$$\delta\ddot{\theta} + 3H\delta\dot{\theta} + 2\left(\frac{\dot{\theta}}{\phi}\delta\dot{\phi} + \frac{\dot{\phi}}{\phi}\delta\dot{\theta} - \frac{\dot{\phi}\dot{\theta}}{\phi^2}\delta\phi\right) - \frac{1}{a^2}\Delta\delta\theta = 0. \quad (19)$$

To find the instability bands, let us write the perturbations as

$$\delta\phi(\mathbf{x}, t) = \delta\phi_0 e^{S(t) + i\mathbf{k}\cdot\mathbf{x}}, \quad (20)$$

$$\delta\theta(\mathbf{x}, t) = \delta\theta_0 e^{S(t) + i\mathbf{k}\cdot\mathbf{x}}, \quad (21)$$

where $\delta\phi_0$ and $\delta\theta_0$ are constants. Inserting these forms into Eqs. (16), (17), (18), and (19), we have the following dispersion relation,

$$\left[\dot{S}^2 + \left(3H + \frac{2\dot{\phi}}{\phi}\right)\dot{S} + \frac{k^2}{a^2}\right] \left[\dot{S}^2 + 3H\dot{S} + \frac{k^2}{a^2} - \omega^2 + V''\right] + 4\omega^2\dot{S}\left(\dot{S} - \frac{\dot{\phi}}{\phi}\right) = 0, \quad (22)$$

where $k^2 \equiv |\mathbf{k}|^2$ and $\omega \equiv \dot{\theta}$. The fluctuations with the momentum \mathbf{k} grow exponentially when the condition

$$\frac{k^2}{a^2} + V'' - \omega^2 < 0, \quad (23)$$

is satisfied. Here we have assumed the inflaton oscillation dominated era. The wavenumber of the maximal growth mode is given by

$$\frac{k_{\max}^2}{a^2} = \frac{1}{16\omega^2}(7\omega^4 - 6V''\omega^2 - V''^2), \quad (24)$$

and the fastest growing rate β_{gr} is given by

$$\beta_{\text{gr}} \equiv \dot{S} = \frac{1}{4} \left| \frac{\omega^2 - V''}{\omega} \right| < \frac{k_{\max}}{a}. \quad (25)$$

Finally, such growing fluctuations go nonlinear to form Q -balls. The Hubble parameter H_* at the Q -ball formation may be expressed as

$$H_* = \frac{1}{\alpha}\beta_{\text{gr}}, \quad (26)$$

where $\alpha > 1$ is a numerical factor that represents the duration of the Q -ball formation, and $\alpha \simeq \mathcal{O}(10)$ for the case of a mass term with negative K [18] and $\alpha \simeq \mathcal{O}(1)$ for the case of a logarithmic term [23].

Next, we investigate the properties of Q -balls. The properties of Q -balls are different for different types, and here we concentrate on the case when the oscillation of the AD field is driven by the thermal logarithmic potential,

$$V_{\text{thermal}} \simeq \alpha_g^2 T^4 \log \left(\frac{|\Phi|^2}{T^2} \right). \quad (27)$$

For the gravity or anomaly mediated SUSY-breaking mechanism, this situation can be realized when

$$\alpha_g^2 T_{\text{osc}}^4 \gg m_\phi^2 \phi_{\text{osc}}^2, \quad (28)$$

and for the gauge-mediated SUSY-breaking mechanism, the thermal logarithmic potential dominates when

$$\alpha_g^2 T_{\text{osc}}^4 \gg \begin{cases} \max\{M_F^4, m_{3/2}^2 \phi_{\text{osc}}^2\} & \text{for } \phi_{\text{osc}} > M_S, \\ m_\phi^2 \phi_{\text{osc}}^2 & \text{for } \phi_{\text{osc}} < M_S. \end{cases} \quad (29)$$

The angular velocity of the homogeneous Affleck-Dine field, ω , is determined by the mass scale around ϕ_{osc} and is estimated to be $\sqrt{2}\alpha_g T_{\text{osc}}^2 / \phi_{\text{osc}}$, which yields from Eq. (24) and Eq. (25)

$$\frac{k_{\text{max}}^2}{a^2} \simeq \frac{3}{2} \frac{\alpha_g^2 T_{\text{osc}}^4}{\phi_{\text{osc}}^2} \quad \text{and} \quad \beta_{\text{gr}} \simeq \frac{\alpha_g T_{\text{osc}}^2}{\sqrt{2} \phi_{\text{osc}}}. \quad (30)$$

The Hubble parameter at the Q -ball formation, Eq. (26), becomes

$$H_* \simeq \frac{1}{\alpha} \frac{\alpha_g T_{\text{osc}}^2}{\phi_{\text{osc}}}, \quad (31)$$

and from Eq. (9) and Eq. (11), the temperature at the Q -ball formation is

$$T_* \simeq \alpha^{-1/4} T_{\text{osc}}. \quad (32)$$

Almost all the baryon/lepton charges produced by the AD mechanism are absorbed into Q -balls. Then, the baryon/lepton charge stored in a Q -ball is estimated as

$$Q \simeq \bar{\epsilon} \beta \left(\frac{\phi_{\text{osc}}}{\alpha_g^{1/2} T_{\text{osc}}} \right)^4, \quad (33)$$

where $\bar{\epsilon}$ is related to the ellipticity of the orbit of the AD field, ϵ , as $\bar{\epsilon} = \epsilon$ for $\epsilon > 0.06$ and $\bar{\epsilon} = 0.06$ for $\epsilon < 0.06$, and $\beta \simeq 6 \times 10^{-4}$ [23]. The numerical factor β represents the dilution due to the cosmic expansion during the Q -ball formation. Here one should notice that the total charge trapped by a Q -ball is proportional to the ellipticity ϵ for $\epsilon > 0.06$. On the other hand, it saturates for $\epsilon < 0.06$, since both negatively charged Q -balls and positively charged ones are formed. Other properties of Q -balls are given by,

$$\begin{aligned} R &\simeq \frac{Q^{1/4}}{\sqrt{2}\alpha_g^{1/2} T_{\text{osc}}}, & \omega &\simeq \frac{\sqrt{2}\pi\alpha_g^{1/2} T_{\text{osc}}}{Q^{1/4}}, \\ \phi_Q &\simeq \alpha_g^{1/2} T_{\text{osc}} Q^{1/4}, & E_Q &\simeq \frac{4\pi\sqrt{2}}{3} \alpha_g^{1/2} T_{\text{osc}} Q^{3/4}, \end{aligned} \quad (34)$$

where R is the radius of Q -balls, ϕ_Q is the value of ϕ at the center of the Q -balls, and E_Q is the energy per a Q -ball. From Eq. (34), the average energy density of Q -balls is estimated as

$$\rho_Q^* \simeq E_Q n_Q \simeq \frac{\alpha_g^2 T_{\text{osc}}^4 (\bar{\epsilon}\beta)^{3/4}}{\eta}, \quad (35)$$

where $n_Q \simeq (k_{\text{max}}/a)^3$ is the number density of Q -balls and η is a numerical factor of order unity. From Eqs. (26) and (35), the corresponding density parameter is given by

$$\Omega_Q^* = \frac{\rho_Q^*}{3M_G^2 H_*^2} \simeq \frac{\alpha^2 (\bar{\epsilon}\beta)^{3/4}}{3\eta} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^2. \quad (36)$$

C. The fate of Q -balls

In this subsection, we examine the fate of Q -balls, which is important for estimating the present amplitude of the GWs from the Q -ball formation. Since the dominant contribution to the potential depends on the temperature which changes with the cosmic time, the properties of Q -balls change as well. Moreover, for some temperatures, other contributions can dominate the thermal logarithmic contribution in the potential, which implies that the properties of Q -balls may drastically change and Q -balls may disappear if the dominant contribution of the potential does not allow a Q -ball solution. Finally Q -balls can decay from their surfaces if the energy per charge is smaller than the nucleon mass or the neutrino mass. In the following, we investigate the fate of Q -balls in detail.

1. Transformation of Q -balls

As mentioned above, the potential for the AD field has temperature dependence and so do the properties of Q -balls. The AD-field value at the center of the Q -balls and the energy per a Q -ball are given by

$$\phi_Q(T) \simeq \alpha_g^{1/2} T Q^{1/4} \simeq (\bar{\epsilon}\beta)^{1/4} \phi_{\text{osc}} \frac{T}{T_{\text{osc}}} \propto T, \quad (37)$$

$$E_Q(T) \simeq \frac{4\pi\sqrt{2}}{3} \alpha_g^{1/2} T Q^{3/4} \simeq \frac{4\pi\sqrt{2}}{3} \frac{(\bar{\epsilon}\beta)^{3/4}}{\alpha_g} \left(\frac{\phi_{\text{osc}}}{T_{\text{osc}}} \right) T \propto T. \quad (38)$$

Here we have used Eq. (33). We can see that $\phi_Q(T)$ and $E_Q(T)$ are proportional to the temperature of the thermal plasma and hence decreases with the cosmic time. Therefore, the average energy density of Q -balls is proportional to Ta^{-3} . Since $T \propto a^{-3/8}$ in the inflaton oscillation dominated era, so the energy density of Q -balls decreases in proportion to $Ta^{-3} \propto T^9$:

$$\rho_Q \simeq \frac{\alpha^{9/4} \alpha_g^2 T^9}{\eta T_{\text{osc}}^5} (\bar{\epsilon}\beta)^{3/4}. \quad (39)$$

After reheating, it decreases in proportion to $Ta^{-3} \propto T^4$. Thus, as long as V_{thermal} dominates the potential of Q -balls, Q -balls never dominate the energy density of the Universe.

a. Gravity or anomaly mediated SUSY-breaking model

While V_{thermal} is proportional to T^4 , $V_{\text{grav}} \simeq \frac{1}{2}m_\phi^2\phi_Q^2(T)$ is proportional to T^2 . Thus, in the case of the gravity or anomaly mediated SUSY-breaking model, using Eq. (37), V_{grav} dominates V_{thermal} at the temperature below T_c given by

$$T_c = \frac{(\bar{\epsilon}\beta)^{1/4}}{\alpha_g} m_\phi \frac{\phi_{\text{osc}}}{T_{\text{osc}}}. \quad (40)$$

Once V_{grav} becomes dominant in the potential of the AD field, the properties of Q -balls are changed. Figure 1 shows the schematic form of this transition. As the temperature decreases, V_{thermal} and ϕ_Q also decrease. At the temperature T_c , the dominant contribution to the potential is changed. In the gravity or anomaly mediated SUSY-breaking model with

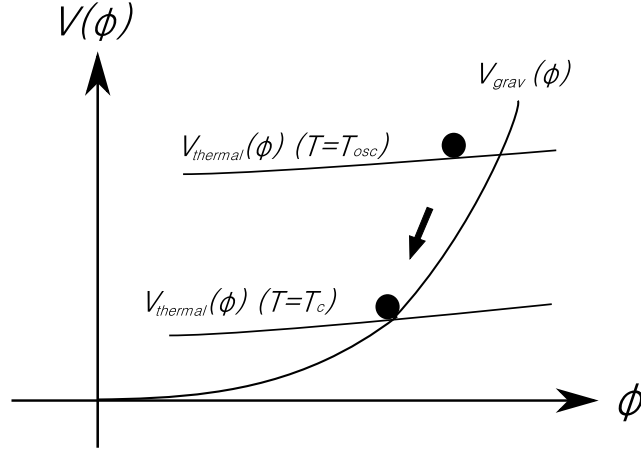


FIG. 1: The scalar potential $V_{\text{grav}} + V_{\text{thermal}}$ is shown. The horizontal axis is the amplitude of the AD field $\phi = \sqrt{2}|\Phi|$. The filled circle represents the position of the AD field. Initially its rotation is driven by V_{thermal} at $T = T_{\text{osc}}(> T_c)$. At the temperature $T = T_c$, V_{grav} dominates over V_{thermal} , and the properties of Q -balls are changed.

negative K , the potential still allows Q -balls but their properties are changed as follows:

$$\begin{aligned} R^2 &\simeq \frac{2}{m_\phi^2|K|}, & \omega &\simeq m_\phi, \\ \phi_Q &\simeq \left(\frac{|K|}{\pi}\right)^{3/4} m_\phi Q^{1/2}, & E_Q &\simeq \frac{1}{4}m_\phi Q. \end{aligned} \quad (41)$$

Here, we have used the fact that Q is conserved through the transition. One should notice that such Q -balls behave like matter because E_Q does not decrease after the transition. In the gravity or anomaly mediated SUSY-breaking model with positive K , on the other hand, the Q -ball solution no longer exists, which makes Q -balls unstable. Therefore the AD field would be almost homogeneous. The energy density of such an AD field also behaves like matter. However, there is an important difference between the cases with positive K and

negative K . While Q -balls can decay only from their surfaces, the almost homogeneous AD field can decay over the whole space. Thus, Q -balls can survive longer and dilute the produced GWs further.

b. Gauge-mediated SUSY-breaking model

In the gauge-mediated SUSY-breaking model, the situations are rather complicated. While V_{thermal} is proportional to T^4 and V_{grav2} is proportional to T^2 , V_{gauge} is independent of T . Thus, there are three possibilities depending on which term dominates the potential next.

- *Case A: V_{gauge} driven Q -ball transformation.*

One is that V_{gauge} dominates V_{thermal} at the critical temperature given by

$$T_c^A = \alpha_g^{-1/2} M_F, \quad (42)$$

so that the type of the Q -ball is changed into the gauge mediation type. The properties of this type of Q -balls are given by

$$\begin{aligned} R &\simeq \frac{Q^{1/4}}{\sqrt{2}M_F}, & \omega &\simeq \frac{\sqrt{2}\pi M_F}{Q^{1/4}}, \\ \phi_Q &\simeq M_F Q^{1/4}, & E_Q &\simeq \frac{4\pi\sqrt{2}}{3} M_F Q^{3/4}. \end{aligned} \quad (43)$$

Note the fact that Q is conserved through the transition. In this case V_{grav2} never becomes the dominant contribution in the potential of the AD field.

- *Case B : $V_{\text{grav}}(K < 0)$ driven Q -ball transformation.*

Another is that V_{grav2} with negative K dominates at the critical temperature given by

$$T_c^B = \frac{(\bar{\epsilon}\beta)^{1/4}}{\alpha_g} m_{3/2} \frac{\phi_{\text{osc}}}{T_{\text{osc}}}, \quad (44)$$

so that the type of the Q -ball is changed into the new type. The properties of this type of Q -balls are given by

$$\begin{aligned} R^2 &\simeq \frac{2}{m_{3/2}^2 |K|}, & \omega &\simeq m_{3/2}, \\ \phi_Q &\simeq \left(\frac{|K|}{\pi} \right)^{3/4} m_{3/2} Q^{1/2}, & E_Q &\simeq \frac{1}{4} m_{3/2} Q. \end{aligned} \quad (45)$$

Here, Q is also conserved at the transition. In this case, V_{gauge} never becomes the dominant contribution in the potential of the AD field.

- *Case C : $V_{\text{grav}}(K > 0)$ driven Q -ball transformation.*

The other one is that V_{grav2} with positive K dominates first at the critical temperature T_c^B given in Eq. (44). This potential does not allow a Q -ball solution and hence the almost homogeneous AD field is recovered. After this transition, the AD field decreases as $\phi \propto H$ because of the cosmic expansion. Thus, V_{gauge} dominates V_{grav2} at

$\phi = \phi_{eq} \equiv M_F^2/m_{3/2}$, which implies that Q -balls are formed again. The properties of this type of Q -balls are those of the delayed type Q -balls and are given by

$$\begin{aligned} R &\simeq \frac{Q^{1/4}}{\sqrt{2}M_F} \simeq (\sqrt{2}m_{3/2})^{-1}, & \omega &\simeq \frac{\sqrt{2}\pi M_F}{Q^{1/4}} \simeq \sqrt{2}\pi m_{3/2}, \\ \phi_Q &\simeq \alpha_g^{1/4} M_F Q^{1/4} \simeq \frac{M_F^2}{m_{3/2}}, & E_Q &\simeq \frac{4\pi\sqrt{2}}{3} M_F Q^{3/4}. \end{aligned} \quad (46)$$

In this case, Q is given by

$$Q \simeq \left(\frac{\phi_{eq}}{M_F} \right)^4 \simeq \left(\frac{M_F}{m_{3/2}} \right)^4. \quad (47)$$

In all cases, Q -balls behave like matter because E_Q does not decrease after the transition and hence Q -balls may dominate the energy density of the Universe. Figure 2 shows the schematic form of the transition of each case. As the temperature decreases, V_{thermal} and ϕ_Q also decrease. The dominant contribution to the potential is changed at the temperature $T_c^{A(B)}$, and hence the type of Q -balls are changed as well.

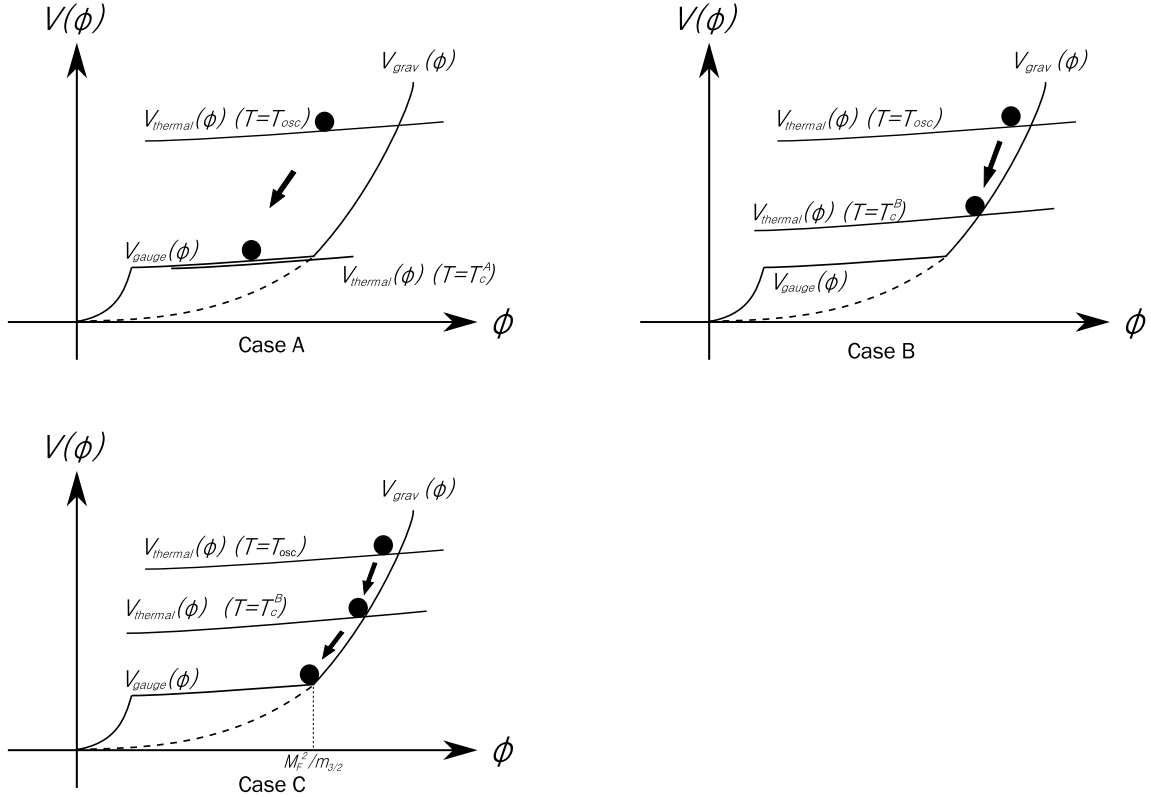


FIG. 2: The scalar potential in the gauge-mediated SUSY-breaking model in each case is shown. The horizontal axis is the amplitude of the AD field. The filled circle represents the position of the AD field. Initially its rotation is driven by V_{thermal} at $T = T_{\text{osc}}$ and dominant potential terms change at $T = T_c^{A(B)}$. There are three possibilities depending on which term dominates afterwards.

2. Decay of Q -balls and AD field

Q -balls can decay into light fermions if the decay processes are kinematically allowed. However, in their interiors the Pauli exclusion principle forbids their decays into fermions [28]. Therefore Q -balls can decay only from their surfaces. This sets the upper bound on the decay rate of Q -balls,

$$\left| \frac{dQ}{dt} \right| \leq \frac{\omega^3 R^2}{48\pi}. \quad (48)$$

In fact, it is almost saturated for the cases we are interested in [28]. Then, we can express the decay rate of Q -balls as

$$\Gamma_{\text{dec}} \equiv \frac{1}{Q} \frac{dQ}{dt} = \frac{\omega^3 R^2}{48\pi Q}, \quad (49)$$

and the Hubble parameter at the Q -ball decay is given by $H_{\text{dec}} = \Gamma_{\text{dec}}$ ¹. The decay rate of the Q -balls is different according to their types. In the case of the gravity or anomaly mediated SUSY-breaking models with negative K , from Eq. (41), the decay rate of Q -balls can be expressed as

$$\Gamma_{\text{dec}} \equiv \frac{1}{Q} \frac{dQ}{dt} \simeq \frac{1}{Q} \frac{m_\phi}{24\pi|K|}. \quad (50)$$

As mentioned above, in the gauge-mediated SUSY-breaking case, there are three possibilities for the fate of Q -balls. From Eqs. (43), (45), (46), and (48), the decay rate can be expressed as

$$\Gamma_{\text{dec}} \equiv \frac{1}{Q} \frac{dQ}{dt} \simeq \begin{cases} \frac{1}{Q^{5/4}} \frac{\pi^2 M_F}{24\sqrt{2}} & \text{for Case A,} \\ \frac{1}{Q} \frac{m_{3/2}}{24\pi|K|} & \text{for Case B,} \\ \frac{\pi^2 m_{3/2}^5}{24\sqrt{2} M_F^4} & \text{for Case C.} \end{cases} \quad (51)$$

The decay temperature should be higher than 1 MeV for the successful big bang nucleosynthesis (BBN), which constrains the parameters of Q -balls, such as ϕ_{osc} and T_R , if Q -balls dominate the energy density of the Universe.² The constraint in each case will be given in the next section.

Note that Q -balls can evaporate when there are thermal plasmas, as discussed in Ref. [29]. Though Q -balls can evaporate away before their decays, this takes place only when Q is small enough, which is unfavorable for our purpose because the initial amplitudes of the produced GWs would also be small. In Appendix A, we will give more quantitative discussion on the charge evaporation from Q -balls.

¹ If there are light bosons and the AD field can decay into them by two-loop interactions, the decay width would be enhanced by a factor $f_s \lesssim 10^3$. However, such bosons do not exist in the context of MSSM because scalar fields other than the flat direction acquire large masses in general, and hence we do not consider such decay processes.

² More precisely, if Q -balls contribute to more than about 1% of the energy density of the Universe, their decay temperature should be higher than 1 MeV for the successful BBN. But, this constraint is not so important for our estimate, and we ignore such small difference.

On the other hand, in the case of the gravity or anomaly mediated SUSY breaking models with positive K , Q -balls vanish at $T = T_c$ and hence the almost homogeneous AD field is recovered, whose amplitude decreases as $\phi \propto H$ due to the cosmic expansion. As long as $h\phi_Q > m_\phi$, the fields coupled with the AD field acquire large masses so that the AD field cannot decay into them. When $h\phi_Q \simeq m_\phi$, the decay into the light fermions is allowed kinematically. The decay rate is given by

$$\Gamma_{\text{dec}} \simeq \frac{h^2}{8\pi} m_\phi. \quad (52)$$

In the case where the AD field dominates the Universe at that time, the Hubble parameter is given by

$$H_{\text{dec}} = \frac{m_\phi^2}{\sqrt{6}hM_G}. \quad (53)$$

Therefore, if $h \gtrsim (m_\phi/M_G)^{1/3} \simeq 10^{-5}$, then, $\Gamma_{\text{dec}} \gtrsim H_{\text{dec}}$, and the AD field decays into light fermions quickly at that time. The decay temperature T_{dec} is estimated as

$$T_{\text{dec}} \simeq \frac{m_\phi}{(6A_T)^{1/4}h^{1/2}} > m_\phi. \quad (54)$$

Thus, the AD field can decay before the BBN and the electroweak symmetry breaking.

Before closing this section, we comment on the decay rate of Q -balls adopted in Ref. [14]. The authors of Ref. [14] considered two possibilities of the Q -ball decay to enhance its decay rate. One is to introduce higher-dimensional operators, which preserve the SUSY but violate the baryon numbers. However, if such operators provide the decays into fermions, the decay rate is strongly constrained by the Pauli blocking as mentioned before. We also would like to point out that it is difficult for such operators to provide the decays into bosons in the context of MSSM, because such bosons acquire large masses through the Yukawa couplings so that the decays into them are kinematically prohibited. Furthermore, even if we find such bosons, the decay rate estimated in Ref. [14] cannot be applied to Q -balls, as pointed out by the authors themselves. The other is the semiclassical decay through the A -terms, which was investigated in detail in Ref. [30]. In Ref. [30], it is found that the instability due to the A -term causes the Q -ball decay in the Minkowski background. Then the authors of Ref. [14] simply apply the expression of the decay rate to the case of the expanding Universe, which is not justified, and conclude that Q -balls quickly decay via this process. In the expanding Universe, however, this process is not effective because the amplitude of the AD field decreases and the A -term becomes soon negligible due to the cosmic expansion. In fact, the authors of Ref. [30] conclude that Q -balls cannot decay via this process in the expanding Universe. Therefore, in this paper, we use the conservative estimate of the decay rate of Q -balls, different from Ref. [14].

III. GRAVITATIONAL WAVES FROM Q -BALLS

In this section we investigate the features of the GWs produced from the Q -ball formation and discuss the prospect for the detection of such GWs. First we estimate the amplitudes and the typical frequencies of the GWs at the Q -ball formation. Then, we evaluate the dilution of the GWs during the subsequent cosmic expansion. As we have seen in the Sec. II C, Q -balls driven by the thermal logarithmic potential never dominate the energy

density of the Universe until their transformation. For this reason, we concentrate on the situation where Q -balls are formed when the thermal logarithmic potential is the dominant contribution to the potential of the AD field. The other cases where Q -balls are produced for the zero-temperature potential are discussed in Appendix B.

A. Generation of GWs

In this subsection, we study the generation of the GWs associated with the fragmentation of the AD field and estimate the amplitudes and frequencies of the GWs. The process of the fragmentation of the AD field is inhomogeneous and nonspherical so that GWs may be emitted at the Q -ball formation.

We evaluate the initial amplitudes and frequencies of the GWs from the Q -ball formation, by using the equations for the transverse-traceless (TT) component of the metric perturbations, following Refs. [12, 14]. By perturbing the Einstein equation, we obtain the equation for the TT component of the metric perturbations,

$$\ddot{h}_{ij}^{\text{TT}}(\mathbf{x}, t) + 3H\dot{h}_{ij}^{\text{TT}}(\mathbf{x}, t) - \frac{\nabla^2}{a^2}h_{ij}^{\text{TT}}(\mathbf{x}, t) = 16\pi G\Pi_{ij}^{\text{TT}}(\mathbf{x}, t), \quad (55)$$

where $h_{ij}^{\text{TT}}(\mathbf{x}, t)$ is the TT component of the metric perturbation and $\Pi_{ij}^{\text{TT}}(\mathbf{x}, t)$ is the TT component of the energy-momentum tensor of the AD field. Instead of using the above Eq. (55), it is easier to use the following equations:

$$\ddot{u}_{ij}(\mathbf{k}, t) + 3H\dot{u}_{ij}(\mathbf{k}, t) + \frac{\mathbf{k}^2}{a^2}u_{ij}(\mathbf{k}, t) = 16\pi GT_{ij}(\mathbf{k}, t). \quad (56)$$

Here $u_{ij}(\mathbf{k}, t)$ and $T_{ij}(\mathbf{k}, t)$ satisfies the relations

$$h_{ij}^{\text{TT}}(\mathbf{k}, t) = \Lambda_{ij,mn}(\hat{\mathbf{k}})u_{mn}(\mathbf{k}, t), \quad (57)$$

$$\Pi_{ij}^{\text{TT}}(\mathbf{k}, t) = \Lambda_{ij,mn}(\hat{\mathbf{k}})T_{mn}(\mathbf{k}, t), \quad (58)$$

$$T_{ij}(\mathbf{x}, t) = \frac{1}{a^2}\partial_i\phi(\mathbf{x}, t)\partial_j\phi(\mathbf{x}, t), \quad (59)$$

where the projection tensor $\Lambda_{ij,mn}$ is defined by

$$\Lambda_{ij,mn}(\hat{\mathbf{k}}) \equiv \left(P_{im}(\hat{\mathbf{k}})P_{jn}(\hat{\mathbf{k}}) - \frac{1}{2}P_{ij}(\hat{\mathbf{k}})P_{mn}(\hat{\mathbf{k}}) \right), \quad (60)$$

$$P_{ij}(\hat{\mathbf{k}}) \equiv \delta_{ij} - \hat{k}_i\hat{k}_j, \quad (61)$$

with $\hat{k}_i \equiv k_i/|\mathbf{k}|$. $h_{ij}^{\text{TT}}(\mathbf{k}, t)$ and $\Pi_{ij}^{\text{TT}}(\mathbf{k}, t)$ are Fourier transforms of $h_{ij}^{\text{TT}}(\mathbf{x}, t)$ and $\Pi_{ij}^{\text{TT}}(\mathbf{x}, t)$, respectively. Since Eq. (56) contains unphysical (gauge) degrees of freedom, we need to follow the time evolution of the variable h_{ij}^{TT} instead of u_{ij} , strictly speaking. However, in the absence of spherical symmetry and homogeneity, it is sufficient to approximate h_{ij}^{TT} by u_{ij} [14].

The energy density of the GWs is given by

$$\rho_{\text{GW}} = \frac{1}{32\pi GL^3} \int d^3\mathbf{x} \dot{h}_{ij}^{\text{TT}}(\mathbf{x}, t)\dot{h}_{ij}^{\text{TT}}(\mathbf{x}, t), \quad (62)$$

which can be approximated as

$$\rho_{\text{GW}} \simeq \frac{1}{32\pi GL^3} \int d^3\mathbf{x} \dot{u}_{ij}(\mathbf{x}, t) \dot{u}_{ij}(\mathbf{x}, t), \quad (63)$$

where $V = L^3$ is the volume of the space, and we have used the fact that the process of fragmentation is nonspherical and $|h_{ij}^{\text{TT}}| \simeq |u_{ij}|$ in this situation.

We now estimate the energy density of the GWs produced by the fragmentation of the AD condensate. For this purpose, we first evaluate the energy-momentum tensor of the AD field T_{ij} . The maximal growth mode of the fluctuation of the AD condensate $\delta\phi$ evolves as

$$\delta\phi(\mathbf{x}, t) = \delta\phi_0 e^{\beta_{\text{gr}} t + i\mathbf{k}_{\text{max}} \cdot \mathbf{x}}, \quad (64)$$

where $\delta\phi_0$ is the initial value of the field perturbation. Then, T_{ij} can be estimated as $T_{ij} \simeq -k_{\text{max}}^2 \delta\phi^2 / 3a^2$ after angle average, and Eq. (56) becomes

$$\ddot{u}_{ij}(2\mathbf{k}_{\text{max}}, t) + \frac{4k_{\text{max}}^2}{a^2} u_{ij}(2\mathbf{k}_{\text{max}}, t) \simeq -\frac{2}{3M_G^2} \frac{k_{\text{max}}^2}{a^2} \delta\phi_0^2 e^{2\beta_{\text{gr}} t}, \quad (65)$$

where the Hubble friction term is neglected since k_{max}/a is larger than H . Since β_{gr} is smaller than k_{max}/a (Eq. (25)), we may use the second term in the left-hand side in Eq. (65) to estimate u_{ij} to give

$$u_{ij} \simeq -\frac{1}{6M_G^2} \delta\phi^2. \quad (66)$$

After a time interval $\Delta t \simeq \ln(\phi_Q/\delta\phi_0)/\beta_{\text{gr}}$, the fluctuations of the flat direction reach $\delta\phi \simeq \phi_Q$, which implies that Q -balls are formed. Therefore, at the last stage of the formation of Q -balls, ρ_{GW} reaches the maximal value, which is estimated using Eqs. (63), (64) and (66) as

$$\rho_{\text{GW}} \simeq \frac{M_G^2}{4} \dot{u}_{ij} \dot{u}_{ij} \simeq \frac{\beta_{\text{gr}}^2}{9M_G^2} \phi_Q^4. \quad (67)$$

The typical frequency of the GWs from the Q -ball formation is given by the wave number of the maximal growing mode, $f_* \simeq k_{\text{max}}/(\pi a)$. The density parameter of the GWs, $\Omega_{\text{GW}}(f_*)$, is then given by

$$\Omega_{\text{GW}}^*(f_*) \simeq \frac{\beta_{\text{gr}}^2 \phi_Q^4}{27M_G^4 H_*^2}, \quad (68)$$

where H_* is the Hubble parameter at the Q -ball formation. The present density parameter and the frequency of the GWs from Q -balls are then given by

$$\Omega_{\text{GW}}^0 = \Omega_{\text{GW}}^* \left(\frac{a_*}{a_0} \right)^4 \left(\frac{H_*}{H_0} \right)^2, \quad (69)$$

$$f_0 = f_* \left(\frac{a_*}{a_0} \right). \quad (70)$$

Therefore, in addition to the redshift factor, the present density parameter contains the dilution factor due to the matter/ Q -balls domination effects.

In the case when the oscillation of the AD field is driven by the thermal logarithmic potential, from Eqs. (30), (31), (33) and (34), the density parameter and the frequency of the GWs at the Q -ball formation are given by

$$\Omega_{\text{GW}}^* = \frac{\alpha^2}{54}(\bar{\epsilon}\beta) \left(\frac{\phi_{\text{osc}}}{M_G} \right)^4, \quad (71)$$

$$f_* = \frac{\sqrt{6}\alpha_g}{2\pi} \frac{T_{\text{osc}}^2}{\phi_{\text{osc}}}. \quad (72)$$

It may be instructive to give alternative derivation of the energy density of the GWs using the quadrupole approximation. By taking the moment of inertia as $I \simeq E_Q R^2 \simeq \phi_Q^2 \omega^2 (k_{\text{max}}/a)^{-5}$ and approximating the time derivative by $\beta_{\text{gr}} \sim k_{\text{max}}/a \sim \omega$, the quadrupole approximation gives the luminosity $L_{\text{GW}} \simeq \ddot{I}^2 M_G^{-2}$ and thus energy density liberated in GWs during the Q -ball formation (the interval $\Delta t \simeq \beta_{\text{gr}}^{-1}$) as

$$\rho_{\text{GW}} \sim \frac{1}{\beta_{\text{gr}} M_G^2} \left(\frac{\phi_Q^2 \beta_{\text{gr}}^3 \omega^2 a^5}{k_{\text{max}}^5} \right)^2 n_Q \sim \frac{\beta_{\text{gr}}^2}{M_G^2} \phi_Q^4, \quad (73)$$

where we have used (30) and (34) and $n_Q \simeq (k_{\text{max}}/a)^3 \sim \beta_{\text{gr}}^3$. Apart from the numerical factor, Eq. (73) nicely coincides with Eq. (67).

B. Cosmic history

In order to evaluate the present amount and typical frequency of the GWs from the Q -ball formation, we need to take into account of the cosmic history after the Q -ball formation. In the case where Q -balls are formed when the thermal logarithmic term is the dominant contribution to the potential of the AD field, there are four possibilities of the cosmic history. They are classified according to the following two conditions. The first condition is whether the Q -ball dominated era exists or not, and it is characterized by H_{dom} and H_{dec} . H_{dom} is the Hubble parameter when the Q -balls (or the AD field) would dominate the energy density of the Universe if such an epoch would exist. H_{dec} is the Hubble parameter when the Q -ball (or the AD field) decays. The second condition is whether the reheating after the inflaton decay occurs before the transformation of the Q -balls due to the change of the effective potential or it occurs after. This condition is characterized by T_R and T_c . Here T_R is the reheating temperature of the inflaton and T_c is temperature at which the zero-temperature potential dominates over the thermal logarithmic potential and hence the properties of the Q -balls are changed. The four cases are shown in Table I. The time evolution of the energy density of each component of the Universe in each case is shown in Fig. 3. The dilution factor of the GWs from Q -ball formation is different in each case.

Case 1	Reheating \Rightarrow Q -ball transformation \Rightarrow Q -ball/AD-field domination \Rightarrow Q -ball/AD-field decay
Case 2	Reheating \Rightarrow Q -ball transformation \Rightarrow Q -ball/AD-field decay (No Q -ball/AD-field domination)
Case 3	Q -ball transformation \Rightarrow Reheating \Rightarrow Q -ball/AD-field domination \Rightarrow Q -ball/AD-field decay
Case 4	Q -ball transformation \Rightarrow Reheating \Rightarrow Q -ball/AD-field decay (No Q -ball/AD-field domination)

TABLE I: The possible cosmic histories.

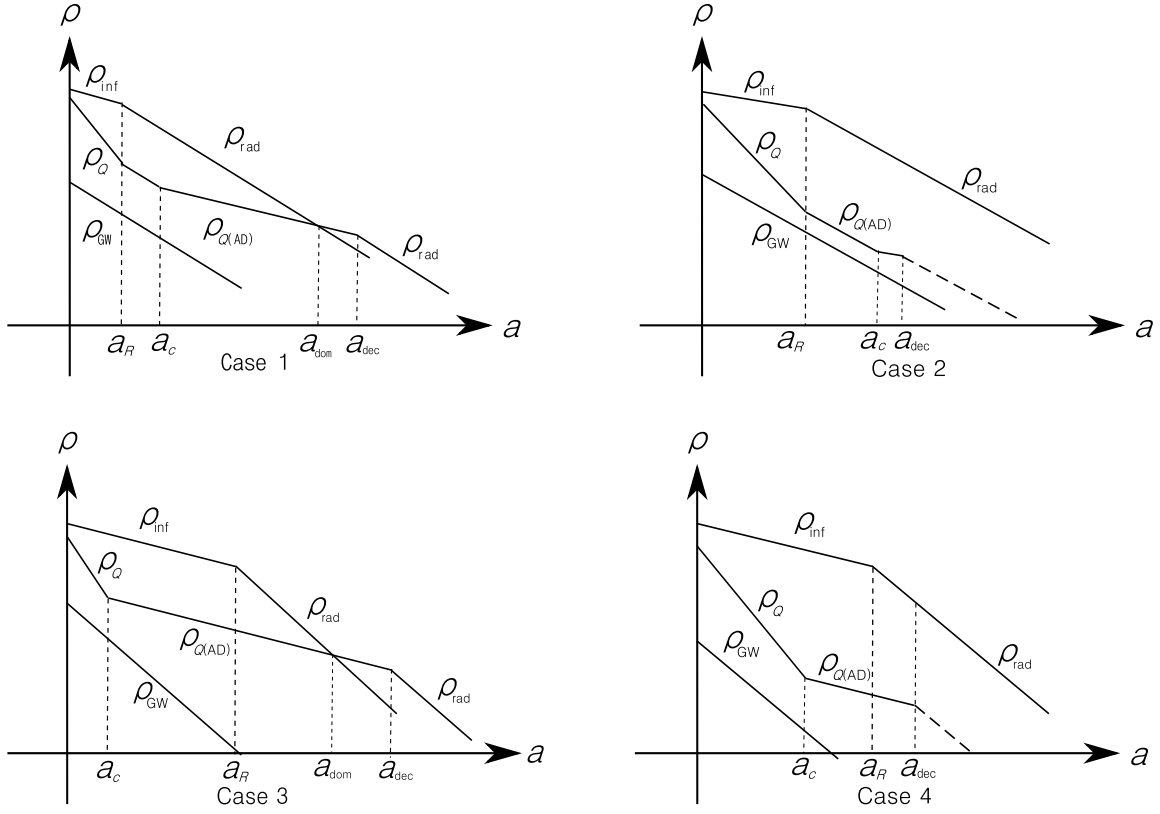


FIG. 3: The schematic time evolution of the energy density of each component (inflaton, radiations from the inflaton decay, Q -balls, radiations from the Q -ball decay, GWs) for each case in Table II is shown. The vertical axis represents the energy density and the horizontal axis represents the scale factor a . The dilution factor of the GWs from the Q -ball formation is different in each case. Even during the inflaton oscillation dominated era, the energy density of the thermal log type Q -balls changes as the temperature of the thermal plasma generated by partial inflaton decay decreases.

We note that the following relations for H_* and T_{osc} , which are derived using Eqs. (9), (11), (27) and (31), are useful in the subsequent discussion,

$$H_* \simeq \frac{\alpha_g^2}{\alpha A_T^{1/2}} \frac{T_R^2 M_G}{\phi_{\text{osc}}^2} \quad \text{and} \quad T_{\text{osc}} \simeq \frac{1}{A_T^{1/4}} \left(\frac{\alpha_g M_G}{\phi_{\text{osc}}} \right)^{1/2} T_R. \quad (74)$$

C. Gravity or anomaly mediated SUSY-breaking model

In this subsection, we evaluate the present amount and the typical frequency of the GWs from the Q -ball formation in the case where the Q -balls are formed when the thermal logarithmic term is the dominant contribution to the potential of the AD field in the gravity or anomaly mediated SUSY-breaking model. In this case, from Eqs. (28) and (74), we have the lower bound on the reheating temperature

$$T_R > \frac{A_T^{1/4}}{\alpha_g} \left(\frac{m_\phi}{M_G} \right)^{1/2} \phi_{\text{osc}} \equiv T_R^c. \quad (75)$$

This bound strongly constrains the frequencies of the produced GWs.

1. Gravity or anomaly mediated SUSY-breaking model with positive K

First, we consider the gravity or anomaly mediated SUSY-breaking model with positive K . In this case, when the zero-temperature potential dominates the effective potential, the Q -balls become unstable and hence decay. Therefore the almost homogeneous AD field is recovered at the critical temperature T_c , Eq. (40), as discussed in Sec. III C.

First of all, we investigate the condition $T_R > T_c$ in detail, which discriminates Cases 1 and 2 ($T_R > T_c$) from Cases 3 and 4 ($T_R < T_c$). Inserting the expression of T_{osc} Eq. (74) into Eq. (40) yields

$$T_c = \frac{A_T^{1/4}}{\alpha_g^{3/2}} (\bar{\epsilon}\beta)^{1/4} \frac{m_\phi \phi_{\text{osc}}^{3/2}}{M_G^{1/2} T_R}, \quad (76)$$

which shows that $T_R > T_c$ is equivalent to

$$T_R > \frac{A_T^{1/8}}{\alpha_g^{3/4}} (\bar{\epsilon}\beta)^{1/8} \frac{m_\phi^{1/2} \phi_{\text{osc}}^{3/4}}{M_G^{1/4}} \equiv T_R^{c1}. \quad (77)$$

Next, in order to examine the condition of the existence of the Q -ball dominated era, we evaluate the Hubble parameter at the would-be Q -ball domination, H_{dom} . In fact, H_{dom} in Case 1 and that of Case 3 coincide with each other. This can be understood as follows. $\Omega_Q \propto T$ during the inflaton oscillation dominated era with thermal log type Q -balls and $\Omega_Q \propto T^{-1}$ during the radiation dominated era with nonthermal type Q -balls, while Ω_Q is constant both during the inflaton oscillation dominated era with zero-temperature type Q -balls and during the radiation dominated era with thermal log type Q -balls. Thus, both for Case 1 and Case 3, the following relation is satisfied:

$$\Omega_Q^{\text{dom}} = \frac{T_R T_c}{T_* T_{\text{dom}}} \Omega_Q^* = 1. \quad (78)$$

Therefore we have

$$H_{\text{dom}} = \frac{A_{\text{dom}}^{1/2}}{M_G} \frac{T_R^2 T_c^2}{T_*^2} \Omega_Q^*, \quad (79)$$

and from Eqs. (32), (36), (74) and (76), we obtain

$$H_{\text{dom}} = \frac{\alpha^{9/2} A_{\text{dom}}^{1/2} A_T}{9\alpha_g^4 \eta^2} (\bar{\epsilon}\beta)^2 \frac{m_\phi^2 \phi_{\text{osc}}^8}{M_G^7 T_R^2}. \quad (80)$$

Here the subscript “dom” indicates that the parameter or variable is evaluated at the would-be Q -ball domination. On the other hand, H_{dec} is given by Eq. (53). Comparing these equations, the condition of the existence of the Q -ball dominated era is equivalent to

$$T_R < \frac{6^{1/4} \alpha^{9/4} h^{1/2} A_{\text{dom}}^{1/4} A_T^{1/2}}{3\eta \alpha_g^2} (\bar{\epsilon}\beta) \frac{\phi_{\text{osc}}^4}{M_G^3} \equiv T_R^{c2}. \quad (81)$$

From T_c in Eq. (76), H_{dom} in Eq. (80), and H_{dec} in Eq. (53), we thus obtain the following conditions on T_R for each cosmic history, which are summarized in Table II.

Case 1	$T_R^{c1}(\phi_{\text{osc}}) < T_R < T_R^{c2}(\phi_{\text{osc}})$
Case 2	$T_R > \max\{T_R^{c1}(\phi_{\text{osc}}), T_R^{c2}(\phi_{\text{osc}})\}$
Case 3	$T_R < \min\{T_R^{c1}(\phi_{\text{osc}}), T_R^{c2}(\phi_{\text{osc}})\}$
Case 4	$T_R^{c2}(\phi_{\text{osc}}) < T_R < T_R^{c1}(\phi_{\text{osc}})$

TABLE II: The conditions of four cases of the cosmic history.

We then calculate the present density parameters of the GWs, Ω_{GW}^0 , and their frequencies, f_0 , for each case.

$$\Omega_{\text{GW}}^0 = D\Omega_{\text{GW}}^*, \quad (82)$$

$$D = \begin{cases} \frac{a_*}{a_R} \frac{a_{\text{dom}}}{a_*} \frac{a_{eq}}{a_0} & (\text{Case 1, 3}), \\ \frac{a_{\text{dec}}}{a_*} \frac{a_{eq}}{a_0} & (\text{Case 2, 4}), \\ \frac{a_R}{a_0} & \end{cases}$$

$$= \begin{cases} \left(\frac{H_R}{H_*}\right)^{2/3} \left(\frac{H_{\text{dec}}}{H_{\text{dom}}}\right)^{2/3} \frac{a_{eq}}{a_0} & (\text{Case 1, 3}), \\ \left(\frac{H_R}{H_*}\right)^{2/3} \frac{a_{eq}}{a_0} & (\text{Case 2, 4}). \end{cases} \quad (83)$$

Here D represents the dilution factor of the GWs due to the matter/ Q -balls domination. The redshift at the Q -ball formation is given by

$$\frac{a_*}{a_0} = \begin{cases} \left(\frac{A_0}{A_T}\right)^{1/3} \left(\frac{A_{\text{dom}}}{A_{\text{dec}}}\right)^{1/12} \frac{T_0}{T_R} \left(\frac{H_{\text{dec}}}{H_{\text{dom}}}\right)^{1/6} \left(\frac{H_R}{H_*}\right)^{2/3} & (\text{Case 1, 3}), \\ \left(\frac{A_0}{A_T}\right)^{1/3} \frac{T_0}{T_R} \left(\frac{H_R}{H_*}\right)^{2/3} & (\text{Case 2, 4}), \end{cases} \quad (84)$$

where the subscript “0” indicates that the parameter is evaluated at present. Here we have neglected the difference between g_* and g_{*s} , where g_{*s} is the relativistic degrees of freedom for the entropy density. a_{eq} and a_0 are the scale factors at the matter-radiation equality and at present, respectively, and so that $a_0/a_{eq} \simeq 3200$. From Eqs. (53), (71), (72), (74), (80), (83), and (84), we thus obtain the present density parameter of the GWs, Ω_{GW}^0 , as

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{18 \cdot 2^{1/3}} \frac{\eta^{4/3} \alpha_g^{4/3}}{\alpha^{1/3} A_{\text{dom}}^{1/3} h^{2/3}} (\bar{\epsilon}\beta)^{-1/3} \left(\frac{T_R}{M_G}\right)^{4/3} \frac{a_{eq}}{a_0} & (\text{Case 1, 3}), \\ \frac{\alpha^{8/3} A_T^{2/3}}{54 \alpha_g^{4/3}} (\bar{\epsilon}\beta) \left(\frac{\phi_{\text{osc}}}{M_G}\right)^{16/3} \frac{a_{eq}}{a_0} & (\text{Case 2, 4}), \end{cases} \quad (85)$$

and the present frequency, f_0 , as

$$f_0 \simeq \begin{cases} \frac{3^{3/4}}{2^{7/12} \pi} \frac{\eta^{1/3} \alpha_g^{4/3} A_0^{1/3}}{\alpha^{1/12} A_{\text{dec}}^{1/12} A_T^{1/3} h^{1/6}} (\bar{\epsilon}\beta)^{-1/3} \frac{M_G^{2/3} T_R^{4/3}}{\phi_{\text{osc}}^2} T_0 & (\text{Case 1, 3}), \\ \frac{\sqrt{6} \alpha_g^{2/3} \alpha^{2/3} A_0^{1/3}}{2\pi A_T^{1/6}} \frac{T_R}{\phi_{\text{osc}}^{2/3} M_G^{1/3}} T_0 & (\text{Case 2, 4}). \end{cases} \quad (86)$$

In Fig. 4 we show the contour plot of the present density parameter of the GWs, Ω_{GW}^0 , and their frequency, f_0 . Although the amplitudes of the GWs can be large, their typical frequencies ($\gtrsim 10^{3\sim 4}$ Hz at least) are too high to be detected by the future experiments, since the sensitivity ranges of future detectors are $\Omega_{\text{GW}}^0 \gtrsim 10^{-7}$ at $f_0 \simeq 10^{2\sim 3}$ Hz for advanced LIGO, $\Omega_{\text{GW}}^0 \gtrsim 10^{-11}$ at $f_0 \simeq 10^{-3\sim -2}$ Hz for LISA, and $\Omega_{\text{GW}}^0 \gtrsim 10^{-16}$ at $f_0 \simeq 10^{-1} \sim 10$ Hz for DECIGO. Large Ω_{GW}^0 requires high reheating temperature and high decay rate, but that results in higher frequency of the GWs in turn.

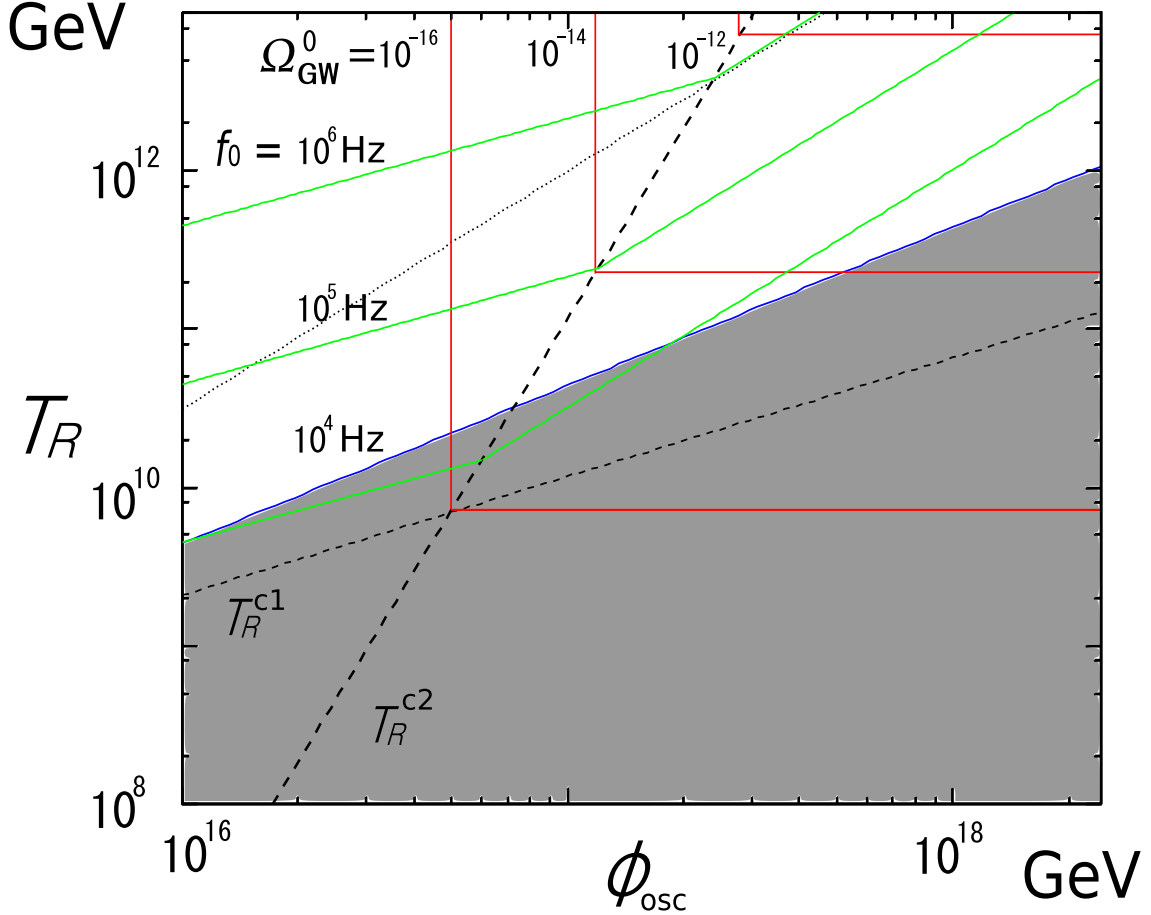


FIG. 4: The contour plot of the present density parameter of the GWs, $\Omega_{\text{GW}}^0 = 10^{-12}, 10^{-14}$ and 10^{-16} (red lines) and its frequency, $f_0 = 10^4$ Hz, 10^5 Hz, and 10^6 Hz (green lines) is shown. Dashed lines represent T_R^{c1} and T_R^{c2} , which distinguish Cases 1-4 (Table II). In the region surrounded by $T = T_R^{c2}$ and $\phi_{\text{osc}} = M_G$, there is the AD-field dominated era. Dotted line represents the boundary of the evaporation of Q -balls, Eq. (88). In the shaded region, thermal log type Q -balls are not formed, $T_R < T_R^c$. The parameters are taken to be $\eta \simeq 1, h \simeq 0.1, \alpha \simeq 1, \alpha_g \simeq 0.1$, and $m_\phi \simeq 10^3$ GeV, and the effective relativistic degrees of freedom of MSSM are assumed for $A_T, A_{\text{dom}}, A_{\text{dec}}$ and A_0 .

In the above argument, we do not consider the possibility that the Q -balls evaporate out via diffusion processes before their decay. However, that could be possible only when charge Q stored in a Q -ball is very small. This can be understood as follows. First one should notice that from Eqs. (33), (74), and (76), the typical charge of the Q -ball and the

transformation temperature are given by

$$Q \simeq 10^{-4} \frac{\phi_{\text{osc}}^6}{M_G^2 T_R^4} \quad \text{and} \quad T_c \simeq \frac{m_\phi \phi_{\text{osc}}^{3/2}}{M_G^{1/2} T_R}. \quad (87)$$

Inserting these values into Eq. (A5) in Appendix A yields

$$\phi_{\text{osc}} \lesssim 10^{8/15} \frac{T_R^{2/3} M_G^{7/15}}{m_\phi^{2/15}} \simeq 10^{15} \text{GeV} \left(\frac{T_R}{10^9 \text{GeV}} \right)^{2/3} \left(\frac{m_\phi}{1 \text{TeV}} \right)^{-2/15} \equiv \phi_{\text{osc}}^{\text{evap}}. \quad (88)$$

In Fig. 4, this constraint is represented by dotted line above which Q -balls evaporate out before their transformation. From Fig. 4, we find that the evaporation is effective at $f_0 \gtrsim 10^6$ Hz for $\Omega_{\text{GW}}^0 > 10^{-16}$. Therefore the charge evaporation from the Q -balls does not change our conclusion.

2. Gravity or anomaly mediated SUSY-breaking model with negative K

Next we discuss the gravity or anomaly mediated SUSY-breaking model with negative K . The only difference between the cases with positive and negative K is that Q -balls do not disappear even after the critical temperature T_c for negative K . Thus, the cosmic history can be classified to four possibilities in the same way as the case with positive K . However, the decay time of the Q -balls, H_{dec} , is different from that of the AD field for positive K . From Eqs. (33), (50), and (74), it can be estimated as

$$H_{\text{dec}} = \frac{\alpha_g^4}{24\pi |K| A_T} (\bar{\epsilon}\beta)^{-1} \frac{M_G^2 T_R^4}{\phi_{\text{osc}}^6} m_\phi. \quad (89)$$

In contrast to the case with positive K , the decay temperature ($A_{\text{dec}}^{-1/4} \sqrt{H_{\text{dec}} M_G}$) can be less than 1 MeV, which is forbidden by the BBN constraint if Q -balls dominate the Universe before their decay. Thus, we have another constraint on the reheating temperature as

$$T_R \gtrsim 0.4 \times 10^{-6} A_T^{1/4} A_{\text{dec}}^{1/8} \left(\frac{|K|(\bar{\epsilon}\beta)}{\alpha_g^4} \right)^{1/4} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^{3/2} \left(\frac{10^3 \text{GeV}}{m_\phi} \right)^{1/4} M_G \equiv T_R^{c, \text{BBN}} \quad (90)$$

for Cases 1 and 3.

Using Eqs. (71), (72), (74), (80), (83), (84) with the decay time Eq. (89), the present density parameter of the GWs, Ω_{GW}^0 , is given by

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{216} \left(\frac{3}{\pi} \right)^{2/3} \frac{\eta^{4/3} \alpha_g^4}{\alpha^{1/3} A_{\text{dom}}^{1/3} A_T^{2/3} |K|^{2/3}} (\bar{\epsilon}\beta)^{-1} \frac{M_G^{2/3} T_R^4}{m_\phi^{2/3} \phi_{\text{osc}}^4} \frac{a_{eq}}{a_0} & \text{(Case 1, 3),} \\ \frac{\alpha^{8/3} A_T^{2/3}}{54 \alpha_g^{4/3}} (\bar{\epsilon}\beta) \left(\frac{\phi_{\text{osc}}}{M_G} \right)^{16/3} \frac{a_{eq}}{a_0} & \text{(Case 2, 4),} \end{cases} \quad (91)$$

and the frequency, f_0 , is given by

$$f_0 \simeq \begin{cases} \frac{\sqrt{3}}{2\pi} \left(\frac{3}{\pi} \right)^{1/6} \frac{\eta^{1/3} \alpha_g^2 A_{\text{dom}}^{1/4} A_0^{1/3}}{\alpha^{1/12} A_{\text{dec}}^{1/12} A_T^{3/4} |K|^{1/6}} (\bar{\epsilon}\beta)^{-1/2} \frac{T_R^2 M_G^{7/6}}{m_\phi^{1/6} \phi_{\text{osc}}^3} T_0 & \text{(Case 1, 3),} \\ \frac{\sqrt{6} \alpha_g^{2/3} \alpha^{2/3} A_0^{1/3}}{2\pi A_T^{1/6}} \frac{T_R}{\phi_{\text{osc}}^{2/3} M_G^{1/3}} T_0 & \text{(Case 2, 4).} \end{cases} \quad (92)$$

From Eqs. (76), (80), and (89), the critical temperatures T_R^{c1} and T_R^{c2} in this case are given by

$$T_R^{c1} = \frac{A_T^{1/8}}{\alpha_g^{3/4}} (\bar{\epsilon}\beta)^{1/8} \frac{m_\phi^{1/2} \phi_{\text{osc}}^{3/4}}{M_G^{1/4}}, \quad (93)$$

$$T_R^{c2} = \left(\frac{8\pi|K|}{3} \right)^{1/6} \frac{\alpha^{3/4} A_{\text{dom}}^{1/12} A_T^{1/3}}{\eta^{1/3} \alpha_g^{4/3}} (\bar{\epsilon}\beta)^{1/2} \frac{m_\phi^{1/6} \phi_{\text{osc}}^{7/3}}{M_G^{3/2}}, \quad (94)$$

which characterize all cases as summarized in Table II.

Figure 5 shows the contour plot of the present density parameter of the GWs, Ω_{GW}^0 , and their frequency, f_0 . Ω_{GW}^0 is independent of the reheating temperature, T_R , in Cases 2 and 4 while it depends on both T_R and ϕ_{osc} in Cases 1 and 3. f_0 is sensitive to T_R . We find that it

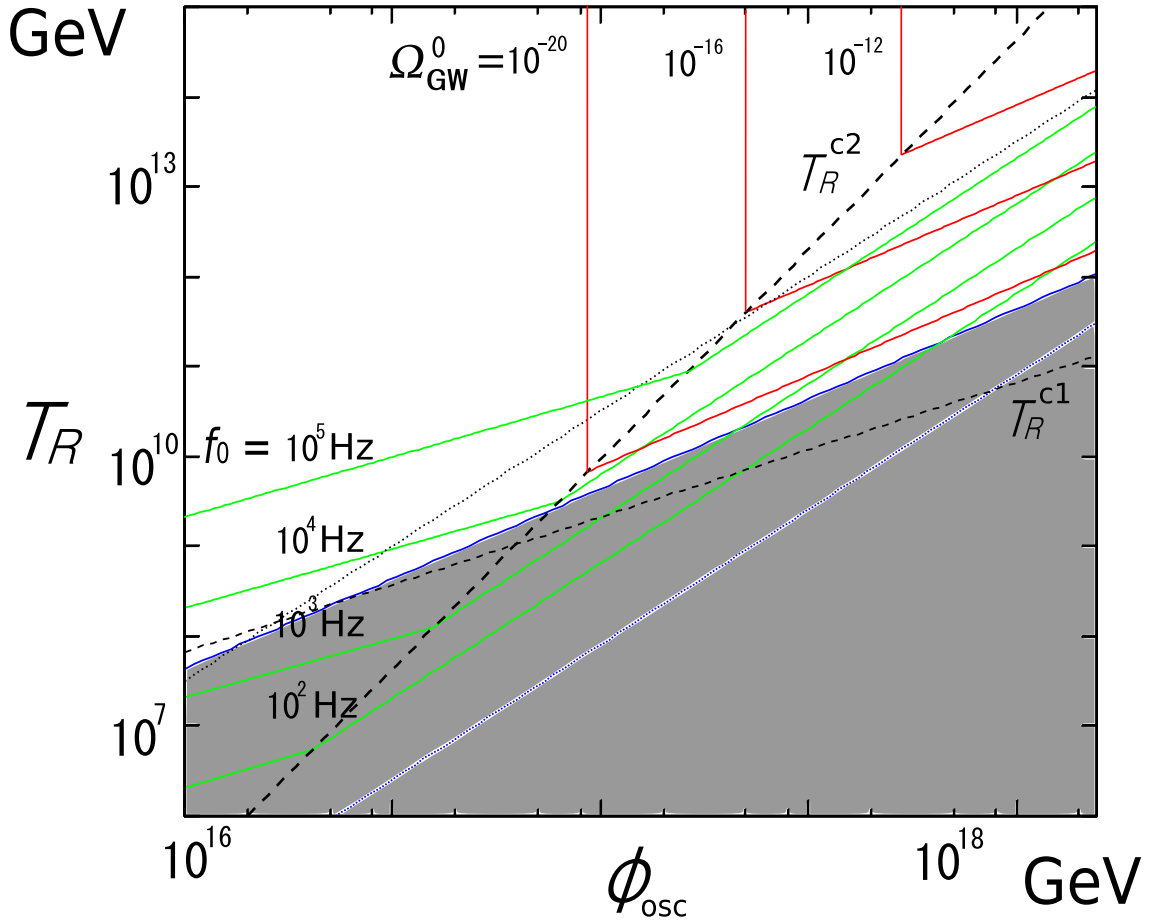


FIG. 5: The same plot as Fig.4 but for the case of the gravity-mediated SUSY-breaking model with negative K is shown. The shaded region represents the forbidden region where $T_R < T_R^c$ or $T_R < T_R^{c,BBN}$. The parameters are taken to be $\eta \simeq 1$, $\alpha \simeq 1$, $\alpha_g \simeq |K| \simeq 0.1$ and $m_\phi \simeq 10^2 \text{ GeV}$, and the effective relativistic degrees of freedom of MSSM are assumed for A_T , A_{dom} , A_{dec} and A_0 .

is almost impossible to detect the GWs from Q -ball formation even by the future detectors in this case, as with the case with positive K . The relatively small decay rate of the Q -balls leads to a lower frequency of the GWs, but it also results in the Q -ball domination, which dilutes GWs.

We comment on the possibility of the Q -ball evaporation. In this case we need to consider the Q -ball evaporation both in the thermal log case [Eq. (A5)] and in the gravity-mediation case [Eq. (A9)]. The condition for the evaporation of the Q -balls before their decay in the case when the thermal logarithmic term dominates the potential is the same as that for positive K . Using Eq. (87), the condition for the Q -ball evaporation after the Q -ball transformation is

$$\phi_{\text{osc}} < 3 \times \frac{T_R^{2/3} M_G^{13/27}}{m_\phi^{4/27}} \simeq 10^{15} \left(\frac{T_R}{10^9 \text{GeV}} \right)^{2/3} \left(\frac{m_\phi}{1 \text{TeV}} \right)^{-4/27} \equiv \phi_{\text{osc}}^{\text{evap}}. \quad (95)$$

As in the case with positive K , we find from Fig. 5 that the evaporation of Q -balls can modify the above estimation of Ω_{GW}^0 and f_0 only when f_0 is too large ($f_0 > 10^5$ Hz). Therefore, even if we take the evaporation effects into account, the conclusion is unchanged.

Summarizing our results in the gravity or anomaly mediated SUSY-breaking models, we find that it is possible to generate a large amount of GWs from the Q -ball formation, but the present frequency is relatively high. This is mainly because the initial frequencies of such GWs are rather large and cannot be redshifted enough. Thus, we conclude that in this mediation mechanism, the GWs from the Q -ball formation cannot be detected by the future detectors (designed or planned) even if thermal log terms induce the Q -ball formation.

D. Gauge-mediated SUSY-breaking model

In this subsection, we study the present properties of the GWs from the Q -ball formation in the case where the Q -balls are formed when the thermal logarithmic term is the dominant contribution to the potential of the AD field in the gauge-mediated SUSY-breaking model.

From the condition that the thermal logarithmic term dominates the potential of the AD field, Eqs. (29), we have the lower bound on the reheating temperature

$$T_R > \max \left\{ \frac{A_T^{1/4}}{\alpha_g} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^{1/2} M_F, A_T^{1/4} \left(\frac{m_{3/2}}{\alpha_g^2 M_G} \right)^{1/2} \phi_{\text{osc}} \right\}, \quad (96)$$

where we have used the relation Eq. (74). This bound strongly constrains the frequencies of the produced GWs. As mentioned before, there are three possibilities of the Q -ball transformation, Cases A, B, and C (Fig. 2), depending on the dominant term (V_{gauge} or V_{grav2}) in the potential and on the sign of K . We estimate the present properties of the GWs from the Q -ball formation in each case.

1. Case A: V_{gauge} driven Q -ball transformation

First we consider Case A, in which the Q -balls are formed by the thermal logarithmic potential and their type is changed into the gauge-mediated type at the critical temperature T_c^A .

From Eqs. (51) and (87), the Hubble parameter at the Q -ball decay is given by

$$H_{\text{dec}} = \frac{\sqrt{2}\pi^2}{48} (\bar{\epsilon}\beta)^{-5/4} \left(\frac{\alpha_g^4}{A_T} \right)^{5/4} \frac{M_G^{5/2} T_R^5 M_F}{\phi_{\text{osc}}^{15/2}}. \quad (97)$$

We have three constraints on the parameters of this model. The first constraint comes from the BBN. For the successful BBN, the Q -balls must decay before BBN and the decay temperature should be larger than 1MeV if Q -balls dominate the energy density of the Universe. This requirement becomes

$$T_R \gtrsim 3 \times 10^{-9} (\bar{\epsilon}\beta)^{1/4} \frac{A_T^{1/4} A_{\text{dec}}^{1/10}}{\alpha_g} \frac{\phi_{\text{osc}}^{3/2}}{M_G^{3/10} M_F^{1/5}} \equiv T_R^{c, \text{BBN}}. \quad (98)$$

The second one is the condition for V_{thermal} to dominate the potential of the AD field Eq. (96),

$$T_R > \frac{A_T^{1/4}}{\alpha_g} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^{1/2} M_F \equiv T_R^{c, \text{th}}. \quad (99)$$

The last one is the condition that $V_{\text{gauge}} > V_{\text{grav2}}$ at $\phi = \phi_c$,

$$T_R > \frac{A_T^{1/4}}{\alpha_g} (\bar{\epsilon}\beta)^{1/4} \frac{m_{3/2} \phi_{\text{osc}}^{3/2}}{M_G^{1/2} M_F} \equiv T_R^{c, \text{gr}}. \quad (100)$$

Here we have used Eqs. (37) and (42).

From Eqs. (32), (36), (42), (74), and (79), we obtain the Hubble parameter at the Q -ball domination as

$$H_{\text{dom}} = \frac{\alpha^{9/2} A_{\text{dom}}^{1/2} A_T^{1/2}}{9\eta^2 \alpha_g^2} (\bar{\epsilon}\beta)^{3/2} \frac{\phi_{\text{osc}}^5 M_F^2}{M_G^6}. \quad (101)$$

The present amount of the GWs from the Q -ball formation is then given by

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{216} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{2/3} \frac{\eta^{4/3} \alpha_g^{10/3}}{\alpha^{1/3} A_{\text{dom}}^{1/3} A_T^{1/2}} (\bar{\epsilon}\beta)^{-5/6} \frac{T_R^{10/3} M_G^{1/3}}{\phi_{\text{osc}}^3 M_F^{2/3}} \frac{a_{eq}}{a_0} & \text{(Case 1, 3),} \\ \frac{\alpha^{8/3} A_T^{2/3}}{54\alpha_g^{4/3}} (\bar{\epsilon}\beta) \left(\frac{\phi_{\text{osc}}}{M_G} \right)^{16/3} \frac{a_{eq}}{a_0} & \text{(Case 2, 4),} \end{cases} \quad (102)$$

and the frequency, f_0 , is estimated as

$$f_0 \simeq \begin{cases} \frac{\sqrt{3}}{2\pi} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{1/6} \frac{\eta^{1/3} \alpha_g^{11/6} A_0^{1/3}}{\alpha^{1/12} A_{\text{dec}}^{1/12} A_T^{11/24}} (\bar{\epsilon}\beta)^{-11/24} \frac{M_G^{13/12} T_R^{11/6}}{\phi_{\text{osc}}^{11/4} M_F^{1/6}} T_0 & \text{(Case 1, 3),} \\ \frac{\sqrt{6} \alpha_g^{2/3} \alpha^{2/3} A_0^{1/3}}{2\pi A_T^{1/6}} \frac{T_R}{\phi_{\text{osc}}^{2/3} M_G^{1/3}} T_0 & \text{(Case 2, 4).} \end{cases} \quad (103)$$

Here we have used Eqs. (71), (72), (74), (83), (84), (97), and (101). The critical temperatures, T_R^{c1} and T_R^{c2} , using Eqs. (42), (97), and (101), can be estimated as

$$T_R^{c1} = \frac{M_F}{\alpha_g^{1/2}}, \quad (104)$$

$$T_R^{c2} = \left(\frac{8\sqrt{2}}{3\pi^2} \right)^{1/5} (\bar{\epsilon}\beta)^{11/20} \frac{\alpha^{9/10} A_T^{7/20} A_{\text{dom}}^{1/10} M_F^{1/5} \phi_{\text{osc}}^{5/2}}{\alpha_g^{7/5} \eta^{2/5} M_G^{17/10}}, \quad (105)$$

which characterize all four cases as summarized in Table II.

The solid (red) line in Fig. 6 represents the maximal amplitudes of the GWs, $\Omega_{\text{GW}}^{0,\text{max}}$, from Q -ball formation at the frequency ranging from 1 Hz to 10^9 Hz in Case A. When $f_0 \lesssim 10^2$ Hz, $\Omega_{\text{GW}}^{0,\text{max}} \propto f_0^{5/2}$, which corresponds to the parameter region with $T_R = T_R^{c,\text{BBN}}$, $10^4 \text{ GeV} < M_F < 10^{10} \text{ GeV}$, and $\phi_{\text{osc}} = M_G$. When $10^2 \text{ Hz} \lesssim f_0 \lesssim 10^8$ Hz, $\Omega_{\text{GW}}^{0,\text{max}} \propto f_0^{20/11}$, which corresponds to the parameter region with $T_R^{c,\text{BBN}} < T_R < T_R^{c2}$, $\phi_{\text{osc}} = M_G$, and $M_F = 10^4$ GeV. Note that in both cases, there is the Q -ball dominated era. When $f_0 \gtrsim 10^8$ Hz, $\Omega_{\text{GW}}^{0,\text{max}}$ is obtained for $\phi_{\text{osc}} = M_G$ without the Q -ball dominated era. As is seen in Fig. 6, $\Omega_{\text{GW}}^{0,\text{max}}$ becomes larger than 10^{-16} but at $f_0 \gtrsim 10^3$ Hz, which goes outside the sensitivity range of the DECIGO or BBO. Therefore, it is almost impossible to detect the GWs from Q -ball formation even by the future detectors in this case.

We comment on the possibility of the Q -ball evaporation. In this case, Q -balls can evaporate out before the decay evaluated above and the estimation of Ω_{GW}^0 and f_0 can break down. However, this is the case only for $f_0 > 10^6$ Hz. Thus, the above conclusion is unchanged by the evaporation effects.

2. Case B: $V_{\text{grav}}(K < 0)$ driven Q -ball transformation

Next we consider Case B. In this case, Q -balls are formed by the thermal logarithmic potential and change their type into the gravity-mediated type ($K < 0$) at the critical temperature T_c^B . This case is similar to the case of the gravity-mediated SUSY-breaking model with negative K . Thus, we need only to replace m_ϕ by $m_{3/2}(< m_\phi)$. While the larger amount of the GWs is obtained by replacing m_ϕ by $m_{3/2}(< m_\phi)$ of Case 3 in Eq. (92), which is favorable for the detection, the present frequency of the GWs is also enhanced, which is unfavorable for the detection.

There are also three constraints on the reheating temperature. The first one is the BBN constraint, which is required when Q -balls dominate the energy density of the Universe, and is given by

$$T_R \gtrsim 2 \times 10^{-6} A_T^{1/4} A_{\text{dec}}^{1/8} \left(\frac{|K|(\bar{\epsilon}\beta)}{\alpha_g^4} \right)^{1/4} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^{3/2} \left(\frac{1 \text{ GeV}}{m_{3/2}} \right)^{1/4} M_G \equiv T_R^{c,\text{BBN}}. \quad (106)$$

This constraint can be derived in the same way as deriving Eq. (90). The second one is the condition that the thermal logarithmic potential should dominate the potential of the AD field at the Q -ball formation and is given by [Eq. (96)],

$$T_R > A_T^{1/4} \left(\frac{m_{3/2}}{\alpha_g^2 M_G} \right)^{1/2} \phi_{\text{osc}} \equiv T_R^{c,\text{th}}. \quad (107)$$

The last one is the condition for V_{grav} to dominate V_{gauge} at the Q -ball transformation,

$$T_R < A_T^{1/4} \alpha_g^{-1} (\bar{\epsilon}\beta)^{1/4} \frac{m_{3/2} \phi_{\text{osc}}^{3/2}}{M_F M_G^{1/2}}. \quad (108)$$

Here we have used Eqs. (37) and (44). Since $m_{3/2}/M_F \simeq M_F/M_G \lesssim 10^{-8}$ if there is no large hierarchy between the SUSY-breaking sector and the messenger sector, the first constraint contradicts with the last one. However, if there is a hierarchy between them, three conditions can be made compatible.

The dashed (green) line in Fig. 6 represents the maximal amount of the GWs, $\Omega_{\text{GW}}^{0,\text{max}}$, from the Q -ball formation at the frequency ranging from 1 Hz to 10^9 Hz in Case B. When $f_0 \lesssim 10^4$ Hz, $\Omega_{\text{GW}}^{0,\text{max}} \propto f_0^{5/2}$, which corresponds to the parameter region with $T_R = T_R^{\text{c,BBN}}$, $m_{3/2} < 10$ GeV, and $\phi_{\text{osc}} = M_G$. For $10^4 \text{ Hz} \lesssim f_0 \lesssim 10^9$ Hz, $\Omega_{\text{GW}}^{0,\text{max}} \propto f_0^{20/11}$, which corresponds to the parameter region with $T_R = T_R^{\text{c,gr}}$, $\phi_{\text{osc}} = M_G$, and $m_{3/2} < 10$ GeV. Note that in both cases, there is the Q -ball dominated era. For $f_0 \gtrsim 10^9$ Hz, $\Omega_{\text{GW}}^{0,\text{max}}$ is obtained for $\phi_{\text{osc}} = M_G$ without the Q -ball dominated era. As seen in Fig. 6, $\Omega_{\text{GW}}^{0,\text{max}}$ is larger than 10^{-16} for $f_0 \gtrsim 10^3$ Hz, which again goes outside the sensitivity range of the DECIGO or BBO. Therefore, it is almost impossible to detect the GWs from Q -ball formation even by the future detectors in this case too. The conclusion is unchanged by the evaporation effects for the same reason discussed in Sec. IIID 1. In this case, the Q -ball evaporation out takes place when $f_0 > 10^{3\sim 4}$ Hz.

3. Case C: $V_{\text{grav}2}(K > 0)$ driven Q -ball transformation

Finally, we consider Case C. In this case, Q -balls are formed by the thermal logarithmic potential and are transformed into the almost homogeneous oscillating AD field at the critical temperature T_c^B . Afterwards, the potential energy of the AD field decreases and V_{gauge} dominates the potential finally. At that time, Q -balls are formed again. Note that the second Q -balls are of the “delayed” type and hence the GWs from the second Q -ball formation cannot be detected, as we will see in Appendix. B.

From Eq. (51), the Hubble parameter at the Q -ball decay is given by

$$H_{\text{dec}} = \frac{\pi^2}{24\sqrt{2}} \frac{m_{3/2}^5}{M_F^4}. \quad (109)$$

We have again three constraints on the parameters of this model. One comes from the BBN constraint,

$$m_{3/2} > 4 \times 10^{-9} A_{\text{dec}}^{1/20} M_F^{4/5} M_G^{1/5} \equiv m_{3/2}^{\text{min}}, \quad (110)$$

which is required when the Q -balls dominate the energy density of the Universe. Another is the condition for V_{thermal} to dominate the potential of the AD field, which coincides with Eq. (107) ,

$$T_R > A_T^{1/4} \left(\frac{m_{3/2}}{\alpha_g^2 M_G} \right)^{1/2} \phi_{\text{osc}} \equiv T_R^{\text{c,th}}. \quad (111)$$

The last one is the condition that $V_{\text{grav}} > V_{\text{gauge}}$ at $\phi = \phi_c$, which coincides with Eq. (108),

$$T_R < \frac{A_T^{1/4}}{\alpha_g} (\bar{\epsilon}\beta)^{1/4} \frac{m_{3/2} \phi_{\text{osc}}^{3/2}}{M_G^{1/2} M_F} \equiv T_R^{\text{c,gr}}. \quad (112)$$

From Eqs. (111) and (112), we have the inequality

$$\phi_{\text{osc}} > (\bar{\epsilon}\beta)^{-1} \frac{M_F^2}{m_{3/2}}. \quad (113)$$

If there is no hierarchy between the SUSY-breaking sector and the messenger sector, $M_F^2 \simeq m_{3/2} M_G$, the inequality (113) cannot be satisfied. Hence we consider the case when there is some hierarchy between them so that Eq. (113) is satisfied.

The Hubble parameter at the Q -ball transformation is given by

$$H_{\text{dom}} \simeq \frac{\alpha^{9/2} A_{\text{dom}}^{1/2} A_T}{9\eta^2 \alpha_g^4} (\bar{\epsilon}\beta)^2 \frac{m_{3/2}^2 \phi_{\text{osc}}^8}{M_G^7 T_R^2}, \quad (114)$$

which is obtained by replacing m_ϕ in Eq. (80) by $m_{3/2}$. The present amount of the GWs from the Q -ball formation is then given by

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{216} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{2/3} \frac{\eta^{4/3} \alpha_g^{4/3}}{\alpha^{1/3} A_{\text{dom}}^{1/3}} (\bar{\epsilon}\beta)^{-1/3} \frac{T_R^{4/3} m_{3/2}^2}{M_G^{2/3} M_F^{8/3}} \frac{a_{eq}}{a_0} & \text{(Case 1, 3),} \\ \frac{\alpha^{8/3} A_T^{2/3}}{54\alpha_g^{4/3}} (\bar{\epsilon}\beta) \left(\frac{\phi_{\text{osc}}}{M_G} \right)^{16/3} \frac{a_{eq}}{a_0} & \text{(Case 2, 4),} \end{cases} \quad (115)$$

and the frequency, f_0 , is estimated as

$$f_0 \simeq \begin{cases} \frac{\sqrt{3}}{2\pi} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{1/6} \frac{\alpha_g^{4/3} A_0^{1/3} \eta^{1/3}}{\alpha^{1/12} A_{\text{dec}}^{1/12} A_T^{1/3} (\bar{\epsilon}\beta)^{1/3}} \frac{T_R^{4/3} m_{3/2}^{1/2} M_G^{5/6}}{\phi_{\text{osc}}^2 M_F^{2/3}} T_0 & \text{(Case 1, 3),} \\ \frac{\sqrt{6} \alpha_g^{2/3} \alpha^{2/3} A_0^{1/3}}{2\pi A_T^{1/6}} \frac{T_R}{\phi_{\text{osc}}^{2/3} M_G^{1/3}} T_0 & \text{(Case 2, 4).} \end{cases} \quad (116)$$

Here we have used Eqs. (71), (72), (74), (83), (84), (109), and (114). From Eqs. (44), (109), and (114), the critical temperatures, T_R^{c1} and T_R^{c2} , are estimated as

$$T_R^{c1} = \left(\frac{A_T (\bar{\epsilon}\beta)}{\alpha_g^6} \right)^{1/8} \frac{m_{3/2}^{1/2} \phi_{\text{osc}}^{3/4}}{M_G^{1/4}}, \quad (117)$$

$$T_R^{c2} = \left(\frac{8\sqrt{2}}{3\pi^2} \right)^{1/2} (\bar{\epsilon}\beta) \frac{\alpha^{9/4} A_{\text{dom}}^{1/4} A_T^{1/2}}{\alpha_g^2 \eta} \frac{M_F^2 \phi_{\text{osc}}^4}{M_G^{7/2} m_{3/2}^{3/2}}, \quad (118)$$

which characterize all four cases as summarized in Table II.

The dotted (blue) line in Fig. 6 is the maximal amplitudes of the GWs, $\Omega_{\text{GW}}^{0,\text{max}}$, from the Q -ball formation at the frequency ranging from 1 Hz to 10^9 Hz in Case C. For $f_0 \lesssim 10^3$ Hz, $\Omega_{\text{GW}}^{0,\text{max}} \propto f_0^{16/7}$, which corresponds to the parameter region with $M_F \simeq 10^4$ GeV, $m_{3/2} < 10$ GeV and $T_R = T_R^{c,\text{th}}$ for $\phi_{\text{osc}} = M_G$. For $10^3 \text{ Hz} \lesssim f_0 \lesssim 10^7 \text{ Hz}$, $\Omega_{\text{GW}}^{0,\text{max}} \propto f_0$, which corresponds to the parameter region with $M_F \simeq 10^4$ GeV, $\phi_{\text{osc}} = M_G$ and $T_R > T_R^{c,\text{th}}$ for $m_{3/2} = 10$ GeV. Note that in both cases, there is the Q -ball dominated era. For $f_0 > 10^7$ Hz, $\Omega_{\text{GW}}^{0,\text{max}}$ is obtained for $\phi_{\text{osc}} = M_G$ and there is no Q -ball dominated era. As seen in Fig. 6, $\Omega_{\text{GW}}^{0,\text{max}}$ is larger than 10^{-16} for $f_0 \gtrsim 10$ Hz, which is on the edge of the DECIGO or BBO sensitivity range. Thus, it is difficult but not impossible to detect such GWs by the next generation detectors. The parameters that realizes $\Omega_{\text{GW}}^0 \simeq 10^{-16}$ at $f_0 \simeq 10$ Hz are given by

$$M_F \simeq 10^4 \text{ GeV}, \quad m_{3/2} \simeq 10 \text{ GeV}, \quad \phi_{\text{osc}} \simeq M_G \quad \text{and} \quad T_R = T_R^{c,\text{th}} \simeq 10^{10} \text{ GeV}. \quad (119)$$

The conclusion above is unchanged by the evaporation effects for the same reason discussed in Sec. IIID1. In this case, the Q -ball evaporation takes place when $f_0 > 10$ Hz. One should notice that though gravitino does not overclose the Universe because of large entropy

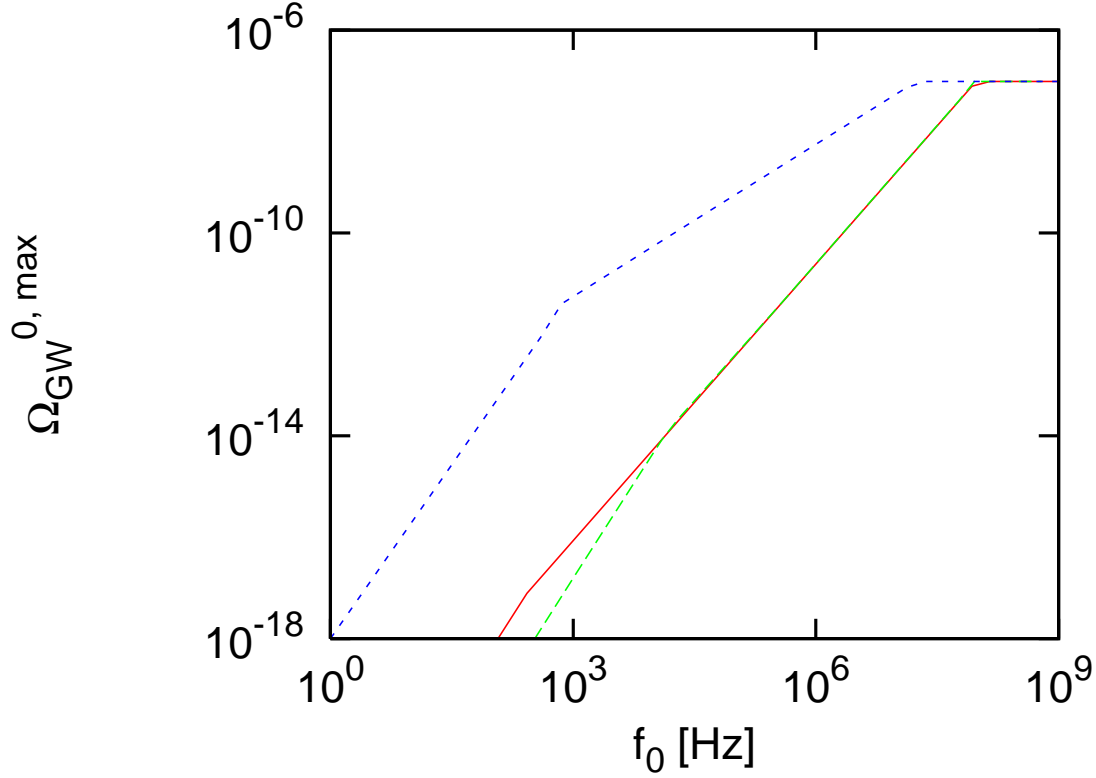


FIG. 6: The maximal amplitude of the GWs from Q -ball formation at the frequency ranging from 1 Hz to 10^9 Hz in each case is shown [Case A (red solid line), Case B (green dashed line) and Case C (blue dotted line)]. The horizontal axis represents the frequency, f_0 , and the vertical axis represents the present density parameter of the GWs, $\Omega_{\text{GW}}^{0, \text{max}}$. The parameters are taken to be $\eta \simeq 1, \alpha \simeq 1$, and $h \simeq \alpha_g \simeq |K| \simeq 0.1$, and the effective relativistic degrees of freedom of MSSM are assumed.

production from Q -ball decay, the next-to-lightest supersymmetric particle (NLSP) decay may spoil the success of the BBN in some cases since the hadronic decay product of NLSP would destroy the light elements [31].

Here we comment on the present baryon and lepton asymmetry. In the situation where the GWs from the Q -ball formation might be detected, both present baryon/lepton asymmetry and radiation are generated by the Q -ball decay. The produced baryon/lepton asymmetry from the Q -ball decay would be rather large and be of the order of unity unless the parameter in the A -term, a_m , is strongly suppressed [32]. In the case of Q -ball with baryonic charge, it is far beyond the experimental bound on the present baryon asymmetry. Even in the case of L -balls, that is, Q -balls with lepton charges but without baryon charge, the situation does not change since the decay temperature of Q -ball is about 500 GeV and hence the lepton asymmetry is converted to baryon asymmetry by sphaleron process [33] that conserves $B - L$ charge. The way to avoid such large baryon/lepton asymmetry in this scenario is that the Q -balls are made of the AD field with $B - L = 0$. In this case, however, we need other baryogenesis mechanisms.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we have discussed the detectability of the GWs from the Q -ball formation. At the Q -ball formation, Q -balls with large Q can produce a large amount of GWs. However, such Q -balls decay slowly and they may dominate the energy density of the Universe so that GWs are significantly diluted. Therefore the detectability of the GWs is determined by these two competing effects.

We have concentrated on the case of thermal log type Q -balls. Such Q -balls do not dominate the energy density of the Universe until the dominant potential for the AD field changes, after that their properties are changed. We have shown that in the gauge-mediated SUSY-breaking model, if the reheating temperature is $T_R \simeq 10^{10}$ GeV and the initial field value of the AD field is $\phi_{\text{osc}} \simeq M_G$ with $m_{3/2} \simeq 10$ GeV and $M_F \simeq 10^4$ GeV, the present density parameter of the GWs from the Q -ball formation can be as large as $\Omega_{\text{GW}}^0 \simeq 10^{-16}$ and their frequency is $f_0 \simeq 10$ Hz. Thus, it is difficult but not impossible to detect them by next-generation gravitational detectors like DECIGO or BBO, but the parameter region for detectable GWs is very small. Moreover, we have shown that it is almost impossible to detect GWs from Q -ball formation in other cases. In other cases when the thermal logarithmic potential drives the Q -ball formation, though the present amounts of the GWs from the Q -ball formation can be as large as $\Omega_{\text{GW}}^0 \simeq 10^{-8}$, the frequencies of such GWs are turned out to be very high. In the cases where zero-temperature potential terms drive the Q -ball formation, the present amount of the GWs from the Q -ball formation is very small due to the large dilution. Thus, the identification of such GWs may determine the decay rate of inflaton or the initial condition of the AD mechanism. In Table III, we show the minimal frequency of the GWs that satisfy $\Omega_{\text{GW}}^0 > 10^{-16}$ in each thermal dominated case, and the maximal amplitudes of GWs in each zero-temperature case are given in Table IV.

Dominant term in the potential	The smallest frequency f_0 that satisfies $\Omega_{\text{GW}}^0 > 10^{-16}$
$V_{\text{thermal}} \Rightarrow V_{\text{grav}}(K > 0)$	10^{3-4} Hz
$V_{\text{thermal}} \Rightarrow V_{\text{grav}}(K < 0)$	10^{2-3} Hz
$V_{\text{thermal}} \Rightarrow V_{\text{grav}}(K > 0) \Rightarrow V_{\text{gauge}}$	10^{1-2} Hz
$V_{\text{thermal}} \Rightarrow V_{\text{gauge}}$	10^{2-3} Hz

TABLE III: The minimal frequency with $\Omega_{\text{GW}}^0 > 10^{-16}$ for thermal log type Q -balls.

Dominant term in the potential	The maximal density parameter Ω_{GW}^0
$V_{\text{grav}}(K < 0)$	10^{-25}
V_{gauge}	10^{-21}
$V_{\text{grav}}(K > 0) \Rightarrow V_{\text{gauge}}$	10^{-24}
$V_{\text{grav}}(K > 0) \Rightarrow V_{\text{thermal}} \Rightarrow V_{\text{gauge}}$	10^{-24}
$V_{\text{thermal}}(c < 0) \Rightarrow V_{\text{gauge}}$	10^{-24}

TABLE IV: The maximal density parameter for Q -balls with the zero-temperature potentials.

We would like to comment on the difficulty in realizing the successful parameter region. One difficulty is the NLSP decay that may spoil the successful BBN when $m_{3/2} \simeq 10$

GeV, since the hadronic energy release from NLSP would cause dissociation process of light elements [31]. However, it can be avoided, for example, if the mass of NLSP is heavy enough to decay quickly. Another is the baryogenesis. One may wonder whether the present baryon asymmetry can be explained simultaneously for such parameter region for the detection of the GWs. Generally speaking, including the present case, the amount of produced baryon asymmetry is typically large for the case that AD condensates or Q -balls (almost) dominate the energy density of the Universe so that the present radiations and baryons are attributed to their decays. This is simply because the number densities of radiations and baryons are of the same order unless the (CP -violating) A -terms are suppressed by some symmetry. Thus, once the GWs from the Q -ball formation are detected, we have the following two possibilities. In the case that such Q -balls are responsible for the present baryon asymmetry, the A -terms are suppressed by symmetry reason. The second option is that Q -balls are irrelevant for baryogenesis, which is realized for the AD fields with $B - L = 0$.

Although the detailed numerical calculations are required to compute the spectrum of GWs associated with Q -ball formation, such GWs may be differentiated from the GWs generated during inflation by their spectrum and from the astrophysical origin like POP III stars by a non-Gaussianity test [34]. On the other hand, a first order phase transition in the early Universe would produce similar spectrum of GWs. However, in our case, the gravitino mass must be around 10 GeV for the detection of the GWs from the Q -ball formation. As mentioned above, such a gravitino mass may induce the NLSP decay problem. Therefore, if collider experiments could determine the gravitino mass by measuring the lifetime of the NLSP [35], that would provide complementary information or even rule out this scenario.

Although the identification is rather difficult, it is true that the detection of such GWs by DECIGO or BBO gives us information of the early Universe and the physics in the high energy scale, for example, it may suggest that the AD mechanism with $B - L = 0$ flat directions is favored.³

Note added in proof: Recently Ref. [41] claims that Q balls may survive even after V_{grav} with $K > 0$ dominates the potential. It is true that a (thin-wall) Q -ball solution exists for such a potential, but it is still unclear whether such a configuration is realized in the expanding Universe. Furthermore, even if this is the case, the dilution factor may become larger so that the detectability of the GWs from Q -ball formation gets even worse. Thus, our conclusions are unchanged.

Acknowledgments

We would like to thank Shinta Kasuya, Masahiro Kawasaki, Fuminobu Takahashi and Jun'ichi Yokoyama for useful comments. This work was partially supported by JSPS (K.K.) and a Grant-in-Aid for Scientific Research from JSPS [No. 20540280 (T.C.) and No. 21740187 (M.Y.)] and from MEXT [No. 20040006 (T.C.)]. This work was also supported in part by Nihon University and by Global COE Program (Global Center of Excellence for Physical Sciences Frontier), MEXT, Japan.

³ Baryogenesis using such flat directions is discussed in the context of the spontaneous baryogenesis mechanism [36].

Appendix A: charge evaporation from Q -balls

As mentioned in Sec. II C, Q -balls can release their charges via the evaporation and the diffusion effects in the thermal bath. In this Appendix, we give the condition that Q -balls are evaporated out by these effects before their decays.

At finite temperature, Q -balls are surrounded by charged free particles. The minimum of free energy is achieved when all charges are distributed in the form of free particles in the thermal plasma, which induces the charge evaporation from the surfaces of Q -balls.

Charge emission of Q -ball consists of the two processes, evaporation at low temperature and diffusion at high temperature. When the difference between the chemical potential of the plasma (μ_p) and that of Q -ball (μ_Q) is small, chemical equilibrium is almost achieved and hence the charge evaporation from the Q -ball surfaces is small. Instead, the charges in the plasmas around the Q -balls are taken away via the diffusion process. The diffusion rate is given by

$$\Gamma_{\text{diff}} \equiv \frac{dQ}{dt} \simeq -4\pi a_{\text{diff}} T. \quad (\text{A1})$$

Here a numerical factor a_{diff} is estimated as $a_{\text{diff}} = 4 - 6$ for quarks and squarks [37, 38], and $a_{\text{diff}} = 100 - 380$ for leptons and sleptons [39].

On the other hand, when the difference between the chemical potential of the plasma and that of Q -ball is large, $\mu_p \ll \mu_Q$, the evaporation process from the Q -ball surface is active. The evaporation rate is estimated as

$$\Gamma_{\text{evap}} \equiv \frac{dQ}{dt} \simeq -4\pi R_Q^2 \xi (\mu_Q - \mu_p) T^2, \quad (\text{A2})$$

where

$$\xi = \begin{cases} 1 & \text{for } T > m_\phi, \\ \left(\frac{T}{m_\phi}\right)^2 & \text{for } T < m_\phi. \end{cases} \quad (\text{A3})$$

Then, depending on the type of Q -ball, we obtain the evaporation rate,

$$\Gamma_{\text{evap}} = \begin{cases} -2\sqrt{2}\pi\xi Q^{1/4} \frac{T}{\alpha_g^{1/4}} & \text{for the thermal log type,} \\ -8\pi\xi \frac{T^2}{m_{\phi(3/2)}|K|} & \text{for the gravity - mediated type,} \\ -2\sqrt{2}\pi\xi Q^{1/4} \frac{T^2}{M_F} & \text{for the gauge - mediated type.} \end{cases} \quad (\text{A4})$$

Here we have used Eqs. (34), (41) and (43) and the fact $\mu_Q \simeq \omega$. The rate of the charge transportation is determined by $\min\{|\Gamma_{\text{evap}}|, |\Gamma_{\text{diff}}|\}$, because both the processes are necessary to strip the charges from the Q -balls. We define the transition temperature as T_{tr} , at which the diffusion process is more effective than the evaporation process. Hereafter we examine the condition for each type that the Q -balls are evaporated out before their decay.

1. Thermal log type

We consider the case where V_{thermal} dominates the potential. Since the Q -ball transformation temperature satisfies $T_c > m_\phi$ in all the cases and hence $\Gamma_{\text{evap}} > \Gamma_{\text{diff}}$ is always satisfied,

we have only to consider the diffusion process. When the condition $H = \frac{1}{Q}|\Gamma_{\text{diff}}|$ is satisfied at $T > T_c$, Q -balls are evaporated out before their transformation, which is realized if the following condition is satisfied:

$$Q < \begin{cases} \frac{4\pi a_{\text{diff}}}{A_T^{1/2}} \frac{T_R^2 M_G}{T_c^3} & \text{for } T_R < T_c, \\ \frac{4\pi a_{\text{diff}}}{A_T^{1/2}} \frac{M_G}{T_c} & \text{for } T_R > T_c. \end{cases} \quad (\text{A5})$$

Here we have used the temperature dependence of the Hubble parameter:

$$H \simeq \begin{cases} \frac{A}{A_T^{1/2}} \frac{T^4}{T_R^2 M_G} & \text{for } T > T_R, \\ A^{1/2} \frac{T^2}{M_G} & \text{for } T_c < T < T_R. \end{cases} \quad (\text{A6})$$

2. Gravity-mediated type

Next we consider the case where $V_{\text{grav}(2)}$ dominates the potential. In this case, the transition temperature T_{tr} is given by

$$T_{\text{tr}} \simeq m_{\phi(3/2)}. \quad (\text{A7})$$

Here we have approximated $|K|a_{\text{diff}} \simeq 1$.

In the case where there is Q -ball dominated era, the Hubble parameter decreases with the temperature as

$$H \simeq \begin{cases} \frac{A}{A_T^{1/2}} \frac{T^4}{T_R^2 M_G} & \text{for } T > T_R, \\ A^{1/2} \frac{T^2}{M_G} & \text{for } T_{\text{dom}} < T < T_R, \\ A^{1/2} \frac{T^{3/2} T_{\text{dom}}^{1/2}}{M_G} & \text{for } T_p < T < T_{\text{dom}}, \\ \frac{A}{A_{\text{dec}}^{1/2}} \frac{T^4}{T_{\text{dec}}^2 M_G} & \text{for } T_{\text{dec}} < T < T_p, \\ A^{1/2} \frac{T^2}{M_G} & \text{for } T < T_{\text{dec}}. \end{cases} \quad (\text{A8})$$

Here $T_p \equiv (T_{\text{dec}}^4 T_{\text{dom}})^{1/5}$. Then $\min\{|\Gamma_{\text{evap}}|, |\Gamma_{\text{diff}}|\}/(QH)$ becomes the highest at $T \simeq m_{\phi(3/2)}$. Thus, the Q -balls are evaporated out before their decays if the following condition is satisfied:

$$Q < \begin{cases} \frac{4\pi a_{\text{diff}} A_T^{1/2}}{A_{\phi(3/2)}} \frac{T_R^2 M_G}{m_{\phi(3/2)}^3} & \text{for } T_R < m_{\phi(3/2)}, \\ \frac{4\pi a_{\text{diff}}}{A_{\phi(3/2)}^{1/2}} \frac{M_G}{m_{\phi(3/2)}} & \text{for } T_{\text{dom}} < m_{\phi(3/2)} < T_R, \\ \frac{4\pi a_{\text{diff}}}{A_{\phi(3/2)}^{1/2}} \frac{M_G}{m_{\phi(3/2)}^{1/2} T_{\text{dom}}^{1/2}} & \text{for } m_{\phi(3/2)} < T_{\text{dom}}. \end{cases} \quad (\text{A9})$$

Here the subscript “ $\phi(3/2)$ ” indicates that A is estimated at $T = m_{\phi(3/2)}$.

3. Gauge-mediated case

Here we consider the case where V_{gauge} dominates the potential. In this case, the transition temperature T_{tr} is given by

$$T_{\text{tr}} \simeq \begin{cases} \sqrt{2}a_{\text{diff}}M_FQ^{-1/4} & \text{for } T_{\text{tr}} > m_\phi, \\ (\sqrt{2}a_{\text{diff}}m_\phi^2M_FQ^{-1/4})^{1/3} & \text{for } T_{\text{tr}} < m_\phi. \end{cases} \quad (\text{A10})$$

These two temperatures coincide for $Q \simeq Q_{\text{cr}} \equiv a^4(M_F/m_\phi)^4$.

Using the similar argument above, we have the condition that the Q -balls are evaporated out before their decays,

$$Q < \begin{cases} \left(\frac{2\sqrt{2}\pi}{A_T^{1/2}} \right)^{4/3} \left(\frac{T_R^2 M_G}{m_\phi^2 M_F} \right)^{4/3} & \text{for } T_R < m_\phi, T_{\text{tr}}, \\ \left(\frac{2\sqrt{2}\pi}{A_T^{1/2}} \right)^{4/3} \left(\frac{M_G}{M_F} \right)^{4/3} & \text{for } T_p, T_{\text{dom}}, m_\phi < T_R, T_{\text{tr}}, \\ \left(\frac{2\sqrt{2}\pi a_{\text{diff}}^{2/3}}{A_T^{1/2}} \right)^{12/11} \left(\frac{M_G}{m_\phi^{2/3} M_F^{1/3}} \right)^{12/11} & \text{for } T_{\text{dec}} < T_{\text{tr}} < m_\phi, T_R, \\ \frac{2\sqrt{2}\pi}{A_{\phi(3/2)}^{1/2}} \frac{T_R^2 M_G}{m_\phi^2 M_F} & \text{for } T_p < m_\phi < T_{\text{dom}} < T_R < T_{\text{tr}}, \\ \left(\frac{2\sqrt{2}\pi}{A_{\text{dec}}^{1/2}} \right)^{4/3} \left(\frac{T_{\text{dec}}^2 M_G}{m_\phi^2 M_F} \right)^{4/3} & \text{for } T_{\text{dec}} < T_{\text{tr}} < T_p. \end{cases} \quad (\text{A11})$$

In conclusion, Q -balls cannot be evaporated out before their decays unless the charge Q inside a Q -ball is small enough.

Appendix B: The case with the zero-temperature potential

In this appendix, we show that the GWs from the Q -ball formation with zero-temperature potential are too small to be detected.

1. Gravity-mediated type

First we consider the gravity-mediated or the “new” type Q -ball. This type is realized for the gravity or anomaly mediated SUSY breaking model with $K < 0$ or for the gauge-mediated SUSY-breaking model with $V_{\text{grav}(2)}(K < 0)$. In this type, the potential is dominated by

$$V_{\text{grav}(2)} \simeq m_{\phi(3/2)}^2 \left[1 + K \log \left(\frac{|\Phi|^2}{M_G^2} \right) \right] |\Phi|^2, \quad (\text{B1})$$

and from Eqs. (24) and (25), the parameters associated with the Q -ball formation are estimates as

$$\beta_{\text{gr}} \simeq \frac{3}{4} m_{\phi(3/2)} |K|, \quad \frac{k_{\text{max}}^2}{a^2} \simeq \frac{3}{2} m_{\phi(3/2)}^2 |K|, \quad \text{and} \quad H_* \simeq \frac{m_{\phi(3/2)} |K|}{2\alpha}. \quad (\text{B2})$$

Here, the factor $\alpha \simeq 30$ represents the dilution due to the cosmic expansion. Then, the amount of the GWs, $\Omega_{\text{GW}}(k_{\text{max}}/(\pi a))$, at the Q -ball formation is estimated from Eq. (68) as

$$\Omega_{\text{GW}}^*(\sqrt{6}m_\phi|K|^{1/2}/(2\pi)) \simeq \frac{3}{16}\alpha^2\tilde{\beta}^2\left(\frac{|K|}{\pi}\right)^3\frac{\phi_{\text{osc}}^4}{M_G^4}. \quad (\text{B3})$$

The baryon or lepton charge stored in a produced Q -ball is estimated as [23]

$$Q \simeq \tilde{\beta}\left(\frac{\phi_{\text{osc}}}{m_{\phi(3/2)}}\right)^2, \quad (\text{B4})$$

where the numerical factor $\tilde{\beta} \simeq 6 \times 10^{-3}$ represents the dilution due to the cosmic expansion. Other properties of the Q -balls are given by

$$\begin{aligned} R^2 &\simeq \frac{2}{m_{\phi(3/2)}^2|K|}, & \omega &\simeq m_{\phi(3/2)}, \\ \phi_Q &\simeq \left(\frac{|K|}{\pi}\right)^{3/4}\tilde{\beta}^{1/2}\phi_{\text{osc}}, & E_Q &\simeq \frac{1}{4}m_{\phi(3/2)}Q. \end{aligned} \quad (\text{B5})$$

Then, the average density of the Q -balls can be estimated as

$$\rho_Q^* \simeq \frac{m_{\phi(3/2)}^2\phi_{\text{osc}}^2\tilde{\beta}}{\eta}. \quad (\text{B6})$$

Here η is a numerical factor of order unity. Now we investigate the present properties of the GWs from the Q -ball formation. One should notice that there is no Q -ball transformation different from the case when the thermal logarithmic potential dominates the potential. Therefore, we have only to consider two cases depending on whether there is a Q -ball dominated era or not. Figure 7 shows the schematic time evolution of the energy density of each component. The Hubble parameters at the Q -ball domination (if any) and the Q -ball decay are given, respectively, by

$$H_{\text{dom}} = \frac{16\alpha^4 A_R^{1/2}\tilde{\beta}^4\phi_{\text{osc}}^4 T_R^2}{9\eta^4|K|^2 M_G^5}, \quad (\text{B7})$$

$$H_{\text{dec}} = \frac{\tilde{\beta}^{-1}m_{\phi(3/2)}^3}{24\pi\phi_{\text{osc}}^2}. \quad (\text{B8})$$

Here we have used Eq. (49) and the relations

$$\Omega_Q^* \simeq \frac{4\alpha^2\tilde{\beta}^2\phi_{\text{osc}}^2}{3\eta^2|K|^2 M_G^2}, \quad (\text{B9})$$

$$H_{\text{dom}} \simeq \Omega_Q^{*2} H_R. \quad (\text{B10})$$

Then, we have the condition for the Q -ball domination,

$$\phi_{\text{osc}} > \left(\frac{3}{128\pi}\right)^{1/6} \frac{\eta^{2/3}|K|^{2/3} M_G^{5/6} m_\phi^{1/2}}{\alpha^{2/3} A_R^{1/12} \tilde{\beta}^{5/6} T_R^{1/3}} \equiv \phi_{\text{osc}}^c. \quad (\text{B11})$$

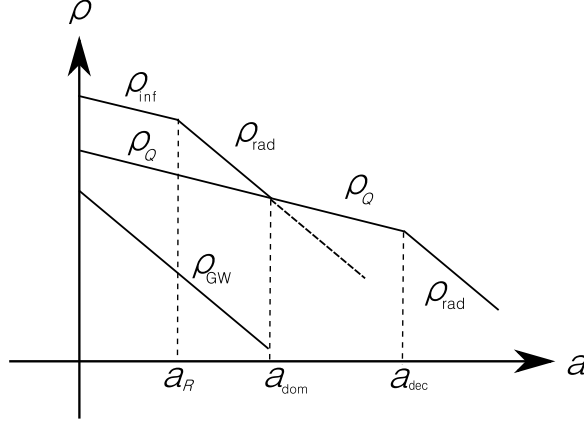


FIG. 7: The time evolution of the energy density of each component is shown schematically. While ρ_{rad} and ρ_{GW} decrease in proportion to a^{-4} , ρ_{inf} in the inflaton oscillation era, and ρ_Q decrease in proportion to a^{-3}

The present density parameter of the GWs from the Q -ball formation in each case is given by

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{192} \left(\frac{3}{\pi} \right)^{2/3} \frac{|K|^5 \tilde{\beta}^{-4/3} \eta^{8/3} m_\phi^{4/3} a_{\text{eq}}}{\pi^3 M_G^{4/3} a_0} & \text{(with the } Q\text{-ball domination),} \\ \frac{2^{2/3}}{12} \frac{\alpha^{8/3} \tilde{\beta}^2 |K|^{7/3} A_R^{1/3} \phi_{\text{osc}}^4 T_R^{4/3} a_{\text{eq}}}{\pi^3 M_G^{14/3} m_\phi^{2/3} a_0} & \text{(without the } Q\text{-ball domination).} \end{cases} \quad (\text{B12})$$

Here we have used Eqs. (83), (B2), (B7) and (B8). Then, from Eq. (B11), we have the upper bound on the present density parameter of GWs,

$$\Omega_{\text{GW}}^0 \leq \frac{1}{192} \left(\frac{3}{\pi} \right)^{2/3} \frac{|K|^5 \tilde{\beta}^{-4/3} \eta^{8/3} m_\phi^{4/3} a_{\text{eq}}}{\pi^3 M_G^{4/3} a_0} \simeq 10^{-25}, \quad (\text{B13})$$

which is too small to be detected by the future detectors.

Here we comment on the possibility of the evaporation of Q -balls before their decay. From Eq. (A9), we find that the Q -balls are evaporated out before their decays if the following condition is satisfied:

$$Q < \frac{4\pi a}{A_{\phi(3/2)}^{1/2}} \frac{M_G}{m_{\phi(3/2)}}, \quad (\text{B14})$$

which yields, using Eq. (B4),

$$\phi_{\text{osc}} < \left(\frac{4\pi a}{A_{\phi(3/2)}^{1/2} \tilde{\beta}} \right)^{1/2} m_{\phi(3/2)}^{1/2} M_G^{1/2} \lesssim 10^{12} \text{GeV}. \quad (\text{B15})$$

Such a small initial field value does not produce significant amounts of GWs and have the present amplitude of GWs is rather small. Thus, our conclusion is unchanged even if we take into account of the possibility that the Q -balls are evaporated out before their decays.

2. Gauge-mediated type

Next we consider the gauge-mediated type Q -balls. In this type, the potential is dominated by

$$V_{\text{gauge}} \simeq M_F^4 \left(\log \frac{|\Phi|^2}{M_S^2} \right)^2. \quad (\text{B16})$$

This potential is quite similar to the thermal logarithmic potential. Then, the parameters associated with the Q -ball formation are given by

$$\frac{k_{\text{max}}^2}{a^2} \simeq \frac{3}{2} \frac{M_F^4}{\phi_{\text{osc}}^2}, \quad \beta_{\text{gr}} \simeq \frac{M_F^2}{\sqrt{2}\phi_{\text{osc}}} \quad \text{and} \quad H_* = \frac{1}{\alpha} \frac{M_F^2}{\phi_{\text{osc}}}, \quad (\text{B17})$$

which yield the amount of the GWs $\Omega_{\text{GW}}(k_{\text{max}}/(\pi a))$ at the Q -ball formation from Eq. (68),

$$\Omega_{\text{GW}}^*(\sqrt{6}M_F^2/(2\pi\phi_{\text{osc}})) \simeq \frac{\alpha^2}{54} (\bar{\epsilon}\beta) \frac{\phi_{\text{osc}}^4}{M_G^4}. \quad (\text{B18})$$

Other properties of the produced Q -balls are obtained by replacing $\alpha_g^{1/2} T_{\text{osc}}$ by M_F in Eq. (34).

Now we evaluate the present properties of the GWs from the Q -ball formation. Note that there is no Q -ball transformation in this type and hence the cosmic history depends on whether there is Q -ball dominated era or not, as is the case with the gravity-mediated type (Appendix B 1). The Hubble parameters at the Q -ball domination (if any) and the Q -ball decay are, respectively, given by

$$H_{\text{dom}} = \frac{\alpha^4 \phi_{\text{osc}}^4}{9\eta^2 M_G^4} A_R^{1/2} (\bar{\epsilon}\beta)^{3/2} \frac{T_R^2}{M_G}, \quad (\text{B19})$$

$$H_{\text{dec}} = \frac{\sqrt{2}\pi^2}{48} (\bar{\epsilon}\beta)^{-5/4} \frac{M_F^6}{\phi_{\text{osc}}^5}, \quad (\text{B20})$$

which give the condition for the Q -ball domination,

$$\phi_{\text{osc}} > \left(\frac{3\sqrt{2}\pi^2}{16} \right)^{1/4} (\bar{\epsilon}\beta)^{-11/36} \frac{\eta^{2/9}}{\alpha^{4/9} A_R^{1/18}} \frac{M_F^{2/3} M_G^{5/9}}{T_R^{2/9}} \equiv \phi_{\text{osc}}^c. \quad (\text{B21})$$

Here we have used Eqs. (49) and (B10). Using Eq. (83), the present density parameter of GWs from Q -ball formation is estimated as

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{216} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{2/3} (\bar{\epsilon}\beta)^{-5/6} \alpha^4 \eta^{4/3} \frac{M_F^{8/3}}{M_G^{4/3} \phi_{\text{osc}}^{4/3}} \frac{a_{\text{eq}}}{a_0} & (\text{with the } Q\text{-ball domination}), \\ \frac{\alpha^{8/3}}{54} (\bar{\epsilon}\beta) A_R^{1/3} \frac{\phi_{\text{osc}}^{14/3} T_R^{4/3}}{M_G^{14/3} M_F^{4/3}} \frac{a_{\text{eq}}}{a_0} & (\text{without the } Q\text{-ball domination}). \end{cases} \quad (\text{B22})$$

The condition that $\phi_Q = (\bar{\epsilon}\beta)^{1/4} \phi_{\text{osc}} > M_S = M_F^2/m_\phi$ yields the upper bound on Ω_{GW}^0 ,

$$\Omega_{\text{GW}}^{0,\text{max}} \simeq \frac{1}{216} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{2/3} (\bar{\epsilon}\beta)^{-5/6} \alpha^4 \eta^{4/3} \frac{m_\phi^{4/3}}{M_G^{4/3}} \frac{a_{\text{eq}}}{a_0} \simeq 10^{-21}. \quad (\text{B23})$$

Again, the GWs generated by the formation of the gauge-mediated type Q -balls are too small to be detected by the next-generation GW detectors.

Finally, we give the condition that the Q -balls are evaporated out before their decays,

$$Q \lesssim \left(\frac{M_G}{M_F} \right)^{4/3} \Leftrightarrow \phi_{\text{osc}} \lesssim M_F^{4/3} M_G^{-1/3}, \quad (\text{B24})$$

which yields $\phi_{\text{osc}} \lesssim 10^8$ GeV since $M_F < 10^{10}$ GeV. Such a small initial field value does not produce significant amounts of GWs and hence the present amplitude of GWs is rather small. Thus, our conclusions are unchanged even if we take the Q -ball evaporation into account.

3. Delayed type

Next, we consider the delayed type Q -balls. In the gauge-mediated SUSY-breaking model, the effective potential for the AD fields is given by the summation of V_{grav2} , V_{gauge} , and V_{thermal} . If the AD field starts its oscillation where the potential is dominated by V_{grav2} with $K > 0$, Q -balls are not formed and the AD field falls down along the potential. Then, V_{thermal} or V_{gauge} dominates the potential of the AD field at a critical temperature or a critical field value, which induces the formation of Q -balls, called the delayed type Q -balls. Thus, there are two cases depending which potential term is responsible for the Q -ball formation.

a. V_{gauge} driven Q -ball formation

First we consider the case where V_{gauge} drives the Q -ball formation. In this case, when $\phi = \phi_{eq} \equiv M_F^2/m_{3/2}$, Q -balls are formed. The Hubble parameter at the Q -ball formation is given by

$$H_* \simeq \frac{M_F^2}{\phi_{\text{osc}}}, \quad (\text{B25})$$

where ϕ_{osc} is the field value at the onset of the oscillation of the AD field and we have used the relation $\phi \propto H$ during the oscillation of the AD field. The condition that V_{gauge} dominates the potential before V_{thermal} is given by $T_R < M_F$. From Eqs. (24) and (25), the parameters associated with the Q -ball formation are given by

$$\beta_{\text{gr}} \simeq \frac{m_{3/2}}{\sqrt{2}} \quad \text{and} \quad \frac{k_{\text{max}}^2}{a^2} \simeq m_{3/2}^2, \quad (\text{B26})$$

which yield the amount of the GWs, $\Omega_{\text{GW}}^*(k_{\text{max}}/(\pi a))$, at the Q -ball formation from Eq. (68),

$$\Omega_{\text{GW}}^*(m_{3/2}/\pi) \simeq \frac{\phi_{\text{osc}}^2 M_F^4}{54 M_G^4 m_{3/2}^2}. \quad (\text{B27})$$

The baryon or lepton charges stored in a Q -ball are given by [23]

$$Q \simeq \left(\frac{\phi_{eq}}{M_F} \right)^4 \simeq \left(\frac{M_F}{m_{3/2}} \right)^4. \quad (\text{B28})$$

Here the numerical factor β [defined in (33)] is almost unity since the Q -balls are formed very quickly so that the cosmic expansion is negligible. Other properties of Q -balls are estimated as,

$$\begin{aligned} R &\simeq \frac{Q^{1/4}}{\sqrt{2}M_F} \simeq (\sqrt{2}m_{3/2})^{-1}, & \omega &\simeq \frac{\sqrt{2}\pi M_F}{Q^{1/4}} \simeq \sqrt{2}\pi m_{3/2}, \\ \phi_Q &\simeq M_F Q^{1/4} \simeq \frac{M_F^2}{m_{3/2}}, & E_Q &\simeq \frac{4\pi\sqrt{2}}{3} M_F Q^{3/4}. \end{aligned} \quad (\text{B29})$$

The energy density of Q -balls at the Q -ball formation is given by

$$\rho_Q^* \simeq E_Q(k_{\text{max}}^3/a^3) \simeq M_F^4. \quad (\text{B30})$$

Then, we evaluate the present properties of the GWs from the Q -ball formation. Again, there is no Q -ball transformation and hence the cosmic history depends on whether there is the Q -ball dominated era or not. The Hubble parameters at the Q -ball domination (if any) and the Q -ball decay are, respectively, given by

$$H_{\text{dom}} = \frac{A_R^{1/2} \phi_{\text{osc}}^4 T_R^2}{9M_G^5}, \quad (\text{B31})$$

$$H_{\text{dec}} = \frac{\pi^2}{24\sqrt{2}} \frac{m_{3/2}^5}{M_F^4}. \quad (\text{B32})$$

Here we have used the relations,

$$\Omega_Q^* \simeq \frac{\phi_{\text{osc}}^2}{3M_G^2}, \quad (\text{B33})$$

$$H_{\text{dom}} \simeq \Omega_Q^{*2} H_R. \quad (\text{B34})$$

Thus, the condition for the Q -ball domination is given by

$$\phi_{\text{osc}} > A_R^{-1/8} \left(\frac{3\pi^2}{8\sqrt{2}} \right)^{1/4} \frac{m_{3/2}^{5/4} M_G^{5/4}}{T_R^{1/2} M_F}. \quad (\text{B35})$$

From Eq. (83), the present density parameter of GWs from Q -ball formation is given by

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{216} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{2/3} \frac{m_{3/2}^{4/3}}{M_G^{4/3}} \frac{a_{\text{eq}}}{a_0} & (\text{with the } Q\text{-ball domination}), \\ \frac{A_R^{1/3}}{54} \frac{\phi_{\text{osc}}^{8/3} T_R^{4/3} M_F^{8/3}}{M_G^{14/3} m_{3/2}^2} \frac{a_{\text{eq}}}{a_0} & (\text{without the } Q\text{-ball domination}). \end{cases} \quad (\text{B36})$$

Considering the condition, that $m_{3/2} < 10\text{GeV}$ for the gauge-mediated SUSY-breaking model, we have the constraint on Ω_{GW}^0 ,

$$\Omega_{\text{GW}}^0 < 10^{-24}, \quad (\text{B37})$$

which is too small for the detection. This conclusion applies also for the second Q -ball formation discussed in Sec. III D 3.

Here we comment on the Q -ball evaporation. From Eqs. (A11), the Q -balls are evaporated out before their decays if the following condition is satisfied:

$$Q \lesssim \left(\frac{M_G}{M_F}\right)^{4/3} \Leftrightarrow M_F \lesssim M_G^{1/4} m_{3/2}^{3/4}. \quad (\text{B38})$$

Thus, we have the inequality with respect to the amount of the GWs at the Q -ball formation,

$$\Omega_{\text{GW}}^* < \frac{\phi_{\text{osc}}^2 m_{3/2}}{54 M_G^3} < 10^{-16}. \quad (\text{B39})$$

Note that this amount of GWs is further diluted at least after matter-radiation equality. Therefore we conclude that even if we take into account the possibility that the Q -balls are evaporated out before their decays, the amount of the present GWs is too small to be detected by the next-generation detectors.

b. V_{thermal} driven Q -ball formation

Next we consider the case where V_{thermal} drives the Q -ball formation. In this case, when $T = T_*$ and $\phi_* \simeq \alpha_g T_*^2 / m_{3/2}$, Q -balls are formed. Note that the reheating takes place after the Q -ball formation because we have the following inequality:

$$T_* \simeq \left(\frac{\alpha_g^2}{A_T^2}\right)^{1/4} \left(\frac{M_G}{\phi_{\text{osc}}}\right)^{1/2} T_R > T_R. \quad (\text{B40})$$

In this case, we have two constraints on the reheating temperature. One is the condition that V_{grav2} dominates V_{thermal} at the onset of the AD field oscillation,

$$T_R < \frac{A_T^{1/4}}{\alpha_g} \frac{m_{3/2}^{1/2}}{M_G^{1/2}} \phi_{\text{osc}} \equiv T_R^{\text{c,gr}}. \quad (\text{B41})$$

Another condition is that V_{thermal} dominates V_{gauge} at the Q -ball formation,

$$T_R > \frac{A_T^{1/4}}{\alpha_g} \frac{\phi_{\text{osc}}^{1/2}}{M_G^{1/2}} M_F \equiv T_R^{\text{c,th}}. \quad (\text{B42})$$

The properties associated with the Q -ball formation are given by

$$\beta_{\text{gr}} \simeq \frac{m_{3/2}}{\sqrt{2}} \quad \text{and} \quad \frac{k_{\text{max}}^2}{a^2} \simeq m_{3/2}^2, \quad (\text{B43})$$

which gives the amount of the GWs, $\Omega_{\text{GW}}^*(k_{\text{max}}/(\pi a))$, at the Q -ball formation,

$$\Omega_{\text{GW}}^*(m_{3/2}/\pi) \simeq \frac{\alpha_g^4 T_R^4}{54 A_T M_G^2 m_{3/2}^2}. \quad (\text{B44})$$

Here we have used the following relations:

$$H_* \simeq \alpha_g \frac{T_*^2}{\phi_{\text{osc}}} \simeq \frac{\alpha_g^2}{A_T^{1/2}} \left(\frac{T_R}{\phi_{\text{osc}}}\right)^2 M_G. \quad (\text{B45})$$

The baryon or lepton charges stored in a Q -ball are given by [23],

$$Q \simeq \left(\frac{\phi_*}{\alpha_g^{1/2} T_*} \right)^4 \simeq \frac{\alpha_g^4}{A_T} \left(\frac{M_G}{\phi_{\text{osc}}} \right)^2 \left(\frac{T_R}{m_{3/2}} \right)^4. \quad (\text{B46})$$

Here the numerical factor β is almost unity since the Q -balls are formed very quickly so that the cosmic expansion is negligible at the formation in this case. Other properties of Q -balls are evaluated as

$$\begin{aligned} R &\simeq \frac{Q^{1/4}}{\sqrt{2}\alpha_g^{1/2}T}, & \omega &\simeq \frac{\sqrt{2}\pi\alpha_g^{1/2}T}{Q^{1/4}}, \\ \phi_Q &\simeq \alpha_g^{1/2}TQ^{1/4}, & E_Q &\simeq \frac{4\pi\sqrt{2}}{3}\alpha_g^{1/2}TQ^{3/4}. \end{aligned} \quad (\text{B47})$$

At the Q -ball formation, the energy density of Q -balls is given by

$$\rho_Q^* \simeq \alpha_g^2 T_*^4 \simeq \frac{\alpha_g^4}{A_T} \left(\frac{M_G}{\phi_{\text{osc}}} \right)^2 T_R^4, \quad (\text{B48})$$

and it decreases with the temperature as

$$\rho_Q \propto A^2 T^9. \quad (\text{B49})$$

Next, we follow the cosmic history after the Q -ball formation and evaluate the present properties of the GWs from the Q -ball formation in this case. The properties of Q -balls are changed into those of the gauge-mediated type Q -balls, when V_{gauge} dominates V_{thermal} at $T \simeq T_C \equiv \alpha_g^{-1/2} M_F$. In the same way as the cases discussed in Sec. IIIC, and Sec. IIID, where the thermal logarithmic potential dominates the effective potential, we have four possibilities of the cosmic history after the Q -ball formation. They are characterized by the critical temperatures, T_R^{c1} and T_R^{c2} . In this case, they are summarized in Table V.

Case 1	$T_R > \max\{T_R^{c1}(\phi_{\text{osc}}), T_R^{c2}(\phi_{\text{osc}})\}$
Case 2	$T_R^{c1}(\phi_{\text{osc}}) < T_R < T_R^{c2}(\phi_{\text{osc}})$
Case 3	$T_R^{c2}(\phi_{\text{osc}}) < T_R < T_R^{c1}(\phi_{\text{osc}})$
Case 4	$T_R < \min\{T_R^{c1}(\phi_{\text{osc}}), T_R^{c2}(\phi_{\text{osc}})\}$

TABLE V: The conditions of four cases of the cosmic history

The critical temperatures are given by

$$T_R^{c1} = \alpha_g^{-1/2} M_F, \quad (\text{B50})$$

$$T_R^{c2} = \left(\frac{3\pi^2}{8\sqrt{2}} \right)^{1/5} \frac{\alpha_g^{3/5} A_T^{3/20} m_{3/2} M_G^{7/10}}{A_{\text{dom}}^{1/10} \phi_{\text{osc}}^{1/2} M_F^{1/5}}. \quad (\text{B51})$$

Here we have used the Hubble parameters at the Q -ball domination (if any) and the Q -ball decay given by

$$H_{\text{dom}} \simeq \frac{A_{\text{dom}}^{1/2} A_T^{1/2} \phi_{\text{osc}}^5 M_F^2}{9\alpha_g^2 M_G^6}, \quad (\text{B52})$$

$$H_{\text{dec}} \simeq \frac{\pi^2}{24\sqrt{2}} \frac{A_T^{5/4} \phi_{\text{osc}}^{5/2} m_{3/2}^5 M_F}{\alpha_g^5 M_G^{5/2} T_R^5}, \quad (\text{B53})$$

Then, from Eq. (83) the present density parameter of GWs from Q -ball formation is estimated as

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{216} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{2/3} \frac{\alpha_g^{2/3} A_T^{1/6}}{A_{\text{dom}}^{1/3}} \frac{T_R^{2/3} m_{3/2}^{4/3}}{M_G M_F^{2/3} \phi_{\text{osc}}^{1/3}} \frac{a_{eq}}{a_0} & (\text{Case 1, 3}), \\ \frac{\alpha_g^{8/3} A_T^{4/3}}{54} \frac{T_R^4 \phi_{\text{osc}}^{4/3}}{M_G^{10/3} m_{3/2}^2} \frac{a_{eq}}{a_0} & (\text{Case 2, 4}). \end{cases} \quad (\text{B54})$$

The condition Eq. (B41) yields the upper limit of the present amount of the GWs in Case 1 and Case 3,

$$\Omega_{\text{GW}}^{0,\text{max}} \simeq \frac{1}{216} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{2/3} \frac{A_T^{1/3}}{A_{\text{dom}}^{2/3}} \frac{m_{3/2}^{5/3} \phi_{\text{osc}}^{1/3}}{M_G^{4/3} M_F^{2/3}} \frac{a_{eq}}{a_0} \leq 10^{-24}, \quad (\text{B55})$$

which is too small to be detected. Here we have used the fact that $\alpha_g \simeq 0.1$, $\phi_{\text{osc}} < M_G$, $m_{3/2} < 10\text{GeV}$, $M_F > 10^4\text{GeV}$, and $a_{eq}/a_0 \simeq 3 \times 10^{-4}$. g_* and g_{*s} are evaluated in the context of MSSM. Moreover, in Case 2 and Case 4, the constraint on the reheating temperature $T_R < T_R^{c2}$ strongly constrains the upper bound of Ω_{GW}^0 and hence the present amount of the GWs cannot be larger than that in Case 1 and Case 3. Therefore we conclude that the GWs generated by the formation of this type of Q -balls are too small to be detected by the next-generation GW detectors.

Here we comment on the effect of the Q -ball evaporation. From Eq. (A11), the Q -balls are evaporated out before their decays if the following condition is satisfied:

$$Q \lesssim \left(\frac{M_G}{M_F} \right)^{4/3} \Leftrightarrow T_R < \frac{A_T^{1/4}}{\alpha_g} \frac{\phi_{\text{osc}}^{1/2} m_{3/2}}{M_G^{1/6} M_F^{1/3}}, \quad (\text{B56})$$

which gives the inequality for the amount of the GWs at the Q -ball formation,

$$\Omega_{\text{GW}}^* \lesssim \frac{1}{54} \frac{m_{3/2}^2 \phi_{\text{osc}}^2}{M_G^{8/3} M_F^{4/3}}. \quad (\text{B57})$$

Considering the fact that $m_{3/2} \lesssim 10\text{ GeV}$, $\phi_{\text{osc}} \lesssim M_G$, and $M_F \gtrsim 10^4\text{ GeV}$, we have the upper limit on Ω_{GW}^* ,

$$\Omega_{\text{GW}}^* < 10^{-16}, \quad (\text{B58})$$

which is further diluted at least after matter-radiation equality. Therefore we conclude that even if we take into account the possibility that the Q -balls are evaporated out before their decays, the amount of the present GWs is too small to be detected by the next-generation detectors.

4. Negative thermal log type

Finally, we consider the case with negative thermal logarithmic potential. So far, we assumed that the contribution of the thermal logarithmic potential is positive. However, it is possible for the term to be negative and hence there can be another type of Q -balls [40].

We then consider this type of Q -balls. If the temperature after inflation is sufficiently high, the AD field is trapped in the potential minimum, $\phi \simeq (\alpha_g T^2 M^{n-3})^{1/(n-1)}$, determined

by the balance between the nonrenormalizable F -term and the negative thermal log term, rather than the negative Hubble mass term. In the gravity or anomaly mediated SUSY-breaking model, this potential minimum exists until the thermal correction to the potential turns to the thermal mass term. At that time, the AD field starts oscillating around the origin by the thermal mass term so that it decays quickly. Thus, Q -balls are not formed in this case. On the other hand, in the gauge-mediated SUSY-breaking model, this potential minimum vanishes when $T = \alpha_g^{-1/2} M_F$. In this case, the AD field starts oscillating from $\phi_{\text{osc}} \simeq (M_F^2 T^2 M^{n-3})^{1/(n-1)}$ by V_{gauge} so that Q -balls are formed. Hereafter, we consider such a type of Q -balls.

In this case, the parameters associated with the Q -ball formation are given by,

$$\frac{k_{\text{max}}^2}{a^2} \simeq \frac{3 M_F^4}{2 \phi_{\text{osc}}^2}, \quad \beta_{\text{gr}} \simeq \frac{M_F^2}{\sqrt{2} \phi_{\text{osc}}} \quad \text{and} \quad H_* \simeq \begin{cases} \frac{A_*^{1/2} M_F^2}{\alpha_g M_G} & \text{for } T_R > \alpha_g^{-1/2} M_F, \\ \frac{A_*}{\alpha_g^2 A_R^{1/2}} \frac{M_F^4}{M_G T_R^2} & \text{for } T_R < \alpha_g^{-1/2} M_F. \end{cases} \quad (\text{B59})$$

The properties of the Q -balls are estimated as

$$Q \simeq \left(\frac{\phi_{\text{osc}}}{M_F} \right)^4, \quad \phi_Q \simeq \phi_{\text{osc}}, \quad \text{and} \quad \rho_Q^* \simeq M_F^4. \quad (\text{B60})$$

Then, the amounts of the GWs, Ω_{GW}^* , at the Q -ball formation are given by

$$\Omega_{\text{GW}}^* \simeq \begin{cases} \frac{\alpha_g^2}{54 A_*} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^2 & \text{for } T_R > \alpha_g^{-1/2} M_F, \\ \frac{\alpha_g^2 A_R}{54 A_*^2} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^2 \left(\frac{T_R}{\alpha_g^{-1/2} M_F} \right)^4 & \text{for } T_R < \alpha_g^{-1/2} M_F. \end{cases} \quad (\text{B61})$$

Note that since the Q -balls are formed quickly, there are no dilution factors.

Next, we follow the cosmic history after the Q -ball formation and evaluate the present properties of the GWs from the Q -ball formation. In this case, there are four possibilities of the cosmic history. They are classified by two criterions. One is whether the Q -ball dominated era exists or not and the other depends which first takes place, the reheating after the inflaton decay or the beginning of the oscillation of the AD field.

The Hubble parameters at the Q -ball domination and at the Q -ball decay, H_{dec} are, respectively, given by

$$H_{\text{dom}} \simeq \begin{cases} \frac{\alpha_g^3}{9 A_*^{3/2}} \frac{M_F^2}{M_G} & \text{for } T_R > \alpha_g^{-1/2} M_F, \\ \frac{\alpha_g^8 A_R^{3/2}}{9 A_*^3} \frac{T_R^{10}}{M_G M_F^8} & \text{for } T_R < \alpha_g^{-1/2} M_F, \end{cases} \quad (\text{B62})$$

$$H_{\text{dec}} = \frac{\sqrt{2} \pi^2}{48} \frac{M_F^6}{\phi_{\text{osc}}^5}, \quad (\text{B63})$$

which gives the condition for the Q -ball domination,

$$\phi_{\text{osc}} > \begin{cases} \left(\frac{3\sqrt{2}\pi^2}{16} \right)^{1/5} \frac{A_*^{3/10}}{\alpha_g^{3/5}} M_G^{1/5} M_F^{4/5} & \text{for } T_R > \alpha_g^{-1/2} M_F, \\ \left(\frac{3\sqrt{2}\pi^2}{16} \right)^{1/5} \frac{A_R^{3/5}}{\alpha_g^{8/5} A_*^{3/10}} \frac{M_G^{1/5} M_F^{14/5}}{T_R^2} & \text{for } T_R < \alpha_g^{-1/2} M_F. \end{cases} \quad (\text{B64})$$

The present density parameter of the GWs from the Q -ball formation is given by

$$\Omega_{\text{GW}}^0 \simeq \begin{cases} \frac{1}{216} \left(\frac{3\pi^2}{\sqrt{2}} \right)^{2/3} \frac{M_F^{8/3}}{M_G^{4/3} \phi_{\text{osc}}^{4/3}} \frac{a_{eq}}{a_0} & \text{with the } Q\text{-ball domination,} \\ \frac{\alpha_g^2}{54 A_*} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^2 \frac{a_{eq}}{a_0} & \text{without the } Q\text{-ball domination and } T_R > \alpha_g^{-1/2} M_F, \\ \frac{\alpha_g^2 A_R}{54 A_*^2} \left(\frac{\phi_{\text{osc}}}{M_G} \right)^2 \left(\frac{T_R}{M_F} \right)^{20/3} \frac{a_{eq}}{a_0} & \text{without the } Q\text{-ball domination and } T_R < \alpha_g^{-1/2} M_F. \end{cases} \quad (\text{B65})$$

Then, the upper bound on Ω_{GW}^0 can be obtained by the same consideration as the case discussed in Appendix B 2 and hence are given by $\Omega_{\text{GW}}^{0,\text{max}} \simeq 10^{-24}$. Moreover, the possibility of the Q -ball evaporation does not change the conclusion as is the case in Appendix B 2. Therefore we conclude that the GWs generated by the formation of this type of Q -balls are too small to be detected by the next-generation GW detectors.

We conclude that the GWs from the Q -ball formation can hardly be detected by the future detectors in the case where the oscillation of the AD field is driven by the zero-temperature potential terms.

-
- [1] For a review, see *e.g.*, M. Maggiore, Phys. Rept. **331**, 283 (2000) [arXiv:gr-qc/9909001]; A. Buonanno, arXiv:gr-qc/0303085; M. Giovannini, arXiv:0901.3026 [astro-ph.CO].
 - [2] A. A. Starobinsky, JETP Lett. **30**, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. **30**, 719 (1979)].
 - [3] N. Seto, S. Kawamura and T. Nakamura, Phys. Rev. Lett. **87**, 221103 (2001) [arXiv:astro-ph/0108011]; S. Kawamura *et al.*, Class. Quant. Grav. **23**, S125 (2006).
 - [4] S. Phinney *et al.*, The Big Bang Observer: Direct Detection of Gravitational Waves from the Birth of the Universe to the Present, NASA Mission Concept Study, (2004).
 - [5] D. Baumann *et al.* [CMBPol Study Team Collaboration], AIP Conf. Proc. **1141**, 10 (2009) [arXiv:0811.3919 [astro-ph]].
 - [6] Y. Watanabe and E. Komatsu, Phys. Rev. D **73**, 123515 (2006) [arXiv:astro-ph/0604176]; T. Chiba, Y. Himemoto, M. Yamaguchi and J. Yokoyama, Phys. Rev. D **76**, 043516 (2007) [arXiv:0705.2851 [gr-qc]]; S. Kuroyanagi, T. Chiba and N. Sugiyama, Phys. Rev. D **79**, 103501 (2009) [arXiv:0804.3249 [astro-ph]].
 - [7] K. Nakayama, S. Saito, Y. Suwa and J. Yokoyama, Phys. Rev. D **77**, 124001 (2008) [arXiv:0802.2452 [hep-ph]]; K. Nakayama, S. Saito, Y. Suwa and J. Yokoyama, JCAP **0806**, 020 (2008) [arXiv:0804.1827 [astro-ph]].
 - [8] S. Weinberg, Phys. Rev. D **69**, 023503 (2004) [arXiv:astro-ph/0306304]; A. Mangilli, N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **78**, 083517 (2008) [arXiv:0805.3234 [astro-ph]].

- [9] K. Ichiki, M. Yamaguchi and J. Yokoyama, Phys. Rev. D **75**, 084017 (2007) [arXiv:hep-ph/0611121].
- [10] A. Kosowsky, M. S. Turner and R. Watkins, Phys. Rev. D **45**, 4514 (1992); A. Kosowsky, M. S. Turner and R. Watkins, Phys. Rev. Lett. **69**, 2026 (1992); L. M. Krauss, Phys. Lett. B **284**, 229 (1992); M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D **49**, 2837 (1994) [arXiv:astro-ph/9310044]; R. Apreda, M. Maggiore, A. Nicolis and A. Riotto, Nucl. Phys. B **631**, 342 (2002) [arXiv:gr-qc/0107033]; C. Grojean and G. Servant, Phys. Rev. D **75**, 043507 (2007) [arXiv:hep-ph/0607107]; C. Caprini, R. Durrer and G. Servant, Phys. Rev. D **77**, 124015 (2008) [arXiv:0711.2593 [astro-ph]]; K. Jones-Smith, L. M. Krauss and H. Mathur, Phys. Rev. Lett. **100**, 131302 (2008) [arXiv:0712.0778 [astro-ph]]; S. J. Huber and T. Konstandin, JCAP **0809**, 022 (2008) [arXiv:0806.1828 [hep-ph]]; C. Caprini, R. Durrer, T. Konstandin, G. Servant, R. Durrer, T. Konstandin and G. Servant, Phys. Rev. D **79**, 083519 (2009) [arXiv:0901.1661 [astro-ph]].
- [11] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. D **56**, 653 (1997) [arXiv:hep-ph/9701423]; R. Easther and E. A. Lim, JCAP **0604**, 010 (2006) [arXiv:astro-ph/0601617]; R. Easther, J. T. Giblin and E. A. Lim, Phys. Rev. Lett. **99**, 221301 (2007) [arXiv:astro-ph/0612294]; J. F. Dufaux, A. Bergman, G. N. Felder, L. Kofman and J. P. Uzan, Phys. Rev. D **76**, 123517 (2007) [arXiv:0707.0875 [astro-ph]]; R. Easther, J. T. Giblin and E. A. Lim, Phys. Rev. D **77**, 103519 (2008) [arXiv:0712.2991 [astro-ph]]; J. F. Dufaux, Phys. Rev. Lett. **103**, 041301 (2009) [arXiv:0902.2574 [astro-ph.CO]]; J. F. Dufaux, G. N. Felder, L. Kofman and O. Navros, JCAP **0903**, 001 (2009) [arXiv:0812.2917 [astro-ph]].
- [12] J. Garcia-Bellido, D. G. Figueroa and A. Sastre, Phys. Rev. D **77**, 043517 (2008) [arXiv:0707.0839 [hep-ph]]; J. Garcia-Bellido and D. G. Figueroa, Phys. Rev. Lett. **98**, 061302 (2007) [arXiv:astro-ph/0701014];
- [13] A. Buonanno, G. Sigl, G. G. Raffelt, H. T. Janka and E. Muller, Phys. Rev. D **72**, 084001 (2005) [arXiv:astro-ph/0412277]; Y. Suwa, T. Takiwaki, K. Kotake and K. Sato, Astrophys. J. **665**, L43 (2007) [AIP Conf. Proc. **990**, 142 (2008)] [arXiv:0706.3495 [astro-ph]].
- [14] A. Kusenko and A. Mazumdar, Phys. Rev. Lett. **101**, 211301 (2008) [arXiv:0807.4554 [astro-ph]]; A. Kusenko, A. Mazumdar and T. Multamaki, Phys. Rev. D **79**, 124034 (2009) [arXiv:0902.2197 [astro-ph.CO]].
- [15] S. R. Coleman, Nucl. Phys. B **262**, 263 (1985) [Erratum-ibid. B **269**, 744 (1986)].
- [16] M. Kawasaki and K. Nakayama, JCAP **0702**, 002 (2007) [arXiv:hep-ph/0611320].
- [17] I. Affleck and M. Dine, Nucl. Phys. B **249**, 361 (1985); M. Dine, L. Randall and S. D. Thomas, Nucl. Phys. B **458**, 291 (1996) [arXiv:hep-ph/9507453].
- [18] A. Kusenko, Phys. Lett. B **405**, 108 (1997) [arXiv:hep-ph/9704273]; A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B **418**, 46 (1998) [arXiv:hep-ph/9709492];
- [19] K. Enqvist and J. McDonald, Phys. Lett. B **425**, 309 (1998) [arXiv:hep-ph/9711514]; K. Enqvist and J. McDonald, Nucl. Phys. B **538**, 321 (1999) [arXiv:hep-ph/9803380].
- [20] S. Kasuya and M. Kawasaki, Phys. Rev. D **61**, 041301 (2000) [arXiv:hep-ph/9909509];
- [21] S. Kasuya and M. Kawasaki, Phys. Rev. D **62**, 023512 (2000) [arXiv:hep-ph/0002285];
- [22] S. Kasuya and M. Kawasaki, Phys. Rev. Lett. **85**, 2677 (2000) [arXiv:hep-ph/0006128].
- [23] S. Kasuya and M. Kawasaki, Phys. Rev. D **64**, 123515 (2001) [arXiv:hep-ph/0106119].
- [24] A. Anisimov and M. Dine, Nucl. Phys. B **619**, 729 (2001) [arXiv:hep-ph/0008058].
- [25] T. Gherghetta, C. F. Kolda and S. P. Martin, Nucl. Phys. B **468**, 37 (1996) [arXiv:hep-ph/9510370].
- [26] A. de Gouvea, T. Moroi and H. Murayama, Phys. Rev. D **56**, 1281 (1997)

- [arXiv:hep-ph/9701244].
- [27] E.W. Kolb and M.S. Turner, *The Early*, (Westview Press, Boulder, CO, 1990).
 - [28] A. G. Cohen, S. R. Coleman, H. Georgi and A. Manohar, Nucl. Phys. B **272**, 301 (1986).
 - [29] M. Laine and M. E. Shaposhnikov, Nucl. Phys. B **532**, 376 (1998) [arXiv:hep-ph/9804237].
 - [30] M. Kawasaki, K. Konya and F. Takahashi, Phys. Lett. B **619**, 233 (2005) [arXiv:hep-ph/0504105].
 - [31] J. L. Feng, S. Su and F. Takayama, Phys. Rev. D **70**, 075019 (2004) [arXiv:hep-ph/0404231]; M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B **649**, 436 (2007) [arXiv:hep-ph/0703122].
 - [32] M. Kawasaki, F. Takahashi and M. Yamaguchi, Phys. Rev. D **66**, 043516 (2002) [arXiv:hep-ph/0205101].
 - [33] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155**, 36 (1985).
 - [34] N. Seto, Astrophys. J. **683**, L95 (2008) [arXiv:0807.1151 [astro-ph]].
 - [35] K. Hamaguchi *et al.* Phys. Rev. D **70** (2004) 115007; J. L. Feng and B. T. Smith, Phys. Rev. D **71** (2005) 015004 [Erratum-ibid. D **71** (2005) 019904]; A. De Roeck, *et al.* Eur. Phys. J. C **49** (2007) 1041; K. Hamaguchi *et al.* JHEP **0703** (2007) 046; S. Asai, K. Hamaguchi and S. Shirai, Phys. Rev. Lett. **103**, 141803 (2009) [arXiv:0902.3754 [hep-ph]].
 - [36] T. Chiba, F. Takahashi and M. Yamaguchi, Phys. Rev. Lett. **92**, 011301 (2004) [arXiv:hep-ph/0304102]; F. Takahashi and M. Yamaguchi, Phys. Rev. D **69**, 083506 (2004) [arXiv:hep-ph/0308173].
 - [37] M. Joyce, T. Prokopec and N. Turok, Phys. Rev. D **53**, 2930 (1996) [arXiv:hep-ph/9410281].
 - [38] H. Davoudiasl and E. Westphal, Phys. Lett. B **432**, 128 (1998) [arXiv:hep-ph/9802335].
 - [39] A. E. Nelson, D. B. Kaplan and A. G. Cohen, Nucl. Phys. B **373**, 453 (1992).
 - [40] S. Kasuya, M. Kawasaki and F. Takahashi, Phys. Rev. D **68**, 023501 (2003) [arXiv:hep-ph/0302154].
 - [41] S. Kasuya, arXiv:1002.4032 [hep-ph].